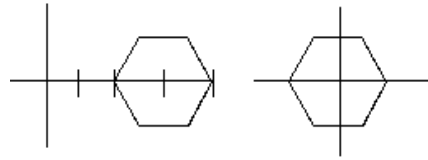


## MOVING A HEXAGON

A regular hexagon  $P_1P_2P_3P_4P_5P_6$  is drawn in the coordinate plane with  $P_1$  at  $(2, 0)$  and  $P_4$  at  $(4, 0)$ . If  $P_n$  is the point  $(x_n, y_n)$ , compute the numerical value of the following product of complex numbers:

$$(x_1 + y_1i)(x_2 + y_2i)(x_3 + y_3i)(x_4 + y_4i)(x_5 + y_5i)(x_6 + y_6i)$$

SOLUTION: Translate the hexagon so that its center is at the origin. Now the vertices represent the roots of  $f(x) = x^6 - 1 = 0$ . Since the  $P_n$  are each 3 more than the roots of  $f(x) = 0$ , they would be the roots of  $f(x - 3) = 0$ , which is  $(x - 3)^6 - 1 = 0$ .



The product of these 6 roots is  $3^6 - 1 = 728$ , the constant term in  $f(x - 3)$ .

### Connections

Complex numbers

Geometry

Translation of roots

Symmetric functions  $(x - a)(x - b)(x - c) = x^3 - (a+b+c)x^2 + (ab+ac+bc)x - abc$

Roots of unity

Can be generalized