ABBE H. HERZIG

WHERE HAVE ALL THE STUDENTS GONE? PARTICIPATION OF DOCTORAL STUDENTS IN AUTHENTIC MATHEMATICAL ACTIVITY AS A NECESSARY CONDITION FOR PERSISTENCE TOWARD THE PH.D.

ABSTRACT. The mathematics community in the U.S. has become concerned about the state of doctoral education, including concerns about high attrition rates and the small numbers of women and students from some racial and ethnic groups. This paper proposes a model of doctoral student persistence and attrition, in which student participation in the life of the department and discipline lead to increased student integration, which is crucial for students' success. Ten faculty members and eighteen graduate students were interviewed about their interests, conceptions, and experiences within mathematics, in a case study of one mathematics department. In this department, students experienced four types of obstacles to their participation: obstacles stemming from the program structure, obstacles to participation in class, obstacles to participating with faculty outside of class, and obstacles stemming from faculty beliefs about teaching and learning. Implications for the retention of mathematics doctoral students are discussed.

KEY WORDS: attrition, doctoral students, graduate students, integration, legitimate peripheral participation, persistence, retention

1. INTRODUCTION

The mathematics community in the U.S. has been concerned with the state of doctoral education, including concerns about the small numbers of women and minorities completing the Ph.D., and the decreasing proportion of Ph.D.s awarded to U.S. citizens.

> Many doctoral students are not prepared to meet undergraduate teaching needs, establish productive research careers, or apply what they have learned in business and industry. This inadequate preparation, continuing high attrition, and the declining interest of domestic students, the inadequate interest of women students, and the near-absent interest of students from underrepresented minorities in doctoral study are problems that transcend the current difficult job market.
>
> (National Research Council, 1992, p. 1)

Estimates of the proportion of doctoral students in mathematics who do not complete the Ph.D. range from 30% to 70% (Bowen and Rudenstine, 1992; Cooper, 2000; Golde, 1996; National Research Council, 1992; Zwick, 1991).

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The large number of doctoral students who leave the educational pipeline after investing several years of their lives represents a range of inefficiently used resources, including the waste in terms of their own lives and self-esteem and the tremendous drain of resources to universities and their funding agencies (Golde, 1999; National Science Foundation, 1998).

Although it might seem that students who leave mathematics are expressing their interests, it is also possible that the mathematics community of practice imposes cultural practices with corresponding implicit expectations on students (Lave and Wenger, 1991) that students may be unable or unwilling to meet. People who succeed in mathematics may be those who are able or willing to adapt themselves to those cultural practices; that is, they learn, or are selected, to work within the existing structure, to play by the existing rules (Stage and Maple, 1996). Individuals whose talents, values, skills, or interests make it difficult or undesirable for them to adapt to that structure may not be able to negotiate the educational and professional systems that are necessary to allow them to do mathematics. The research on the experiences of doctoral students in mathematics may be confounded by the loss of those students without a pre-disposition to accept the epistemological and methodological practices of the discipline. So, it is not clear whether students who leave mathematics do so because they are rejecting the intellectual content of mathematics or some other parts of the sociocultural practices that are part of becoming a mathematician.

Previous research has investigated the role of three types of factors in the persistence and attrition of graduate students: features of the graduate students (for example, gender and race or family responsibilities), features of the institutions of study (for example, financial resources and support, or the structure of the department and program), and features of the academic disciplines (for example, the nature and degree of mentoring and advising, or the isolated nature of graduate study). Notably absent from the research literature are investigations about pedagogical issues and their effects on students.

In the next section, I develop a model of doctoral student attrition and persistence that attempts to explain these diverse 'causes.' I then describe a study designed to investigate the doctoral student experience in mathematics based on predictions from this model.

1.1. *A model of the persistence and attrition of mathematics doctoral students*

Tinto (1993) proposes a model for studying doctoral student attrition, based on Durkheim's (1951; cited in Tinto, 1993) model of suicide, both of which

> represent a form of voluntary withdrawal from local communities that is as much a reflection of the community as it is of the individual who withdraws. Moreover, each can be seen to signal somewhat similar forms of rejection of conventional norms regarding the value of persisting in those communities.
>
> (Tinto, 1993, p. 99)

Tinto argues by analogy that students who do not become integrated in the social and academic communities of their graduate programs are more likely to voluntarily withdraw from graduate school. Experiences that enhance student membership in these communities strengthen the students' goals and commitments, which in turn increase the likelihood that they will persist.

One important mechanism by which students become academically integrated is through the socialization that stems from relationships with department faculty, particularly advisors. A faculty member "serves as a role model and becomes the primary socializing agent in the department.... It is the number of faculty members a student comes to know as professional colleagues that is associated with involvement in the doctoral program" which in turn is "directly related to doctoral degree progress" (Girves and Wemmerus, 1988, p. 185). The underlying process of these interactions can be conceptualized as legitimate peripheral participation (Lave and Wenger, 1991), in which students participate in authentic ways at the periphery of a community of practice, which in time moves them to more central participation. "The important point concerning learning is one of access to practice as resource for learning, rather than to instruction" (Lave and Wenger, 1991, p. 85). 'Old-timers' (i.e. the faculty, or the more advanced graduate students) set the stage for the activity of the newcomers, and students learn through their participation in the activity of the academic community.

Students are also members of other communities, such as family and work. To some extent, persistence depends on the degree to which they successfully negotiate the competing demands of these different communities, or alternatively, the way that these other communities interfere with or enhance student participation and integration in their programs (Tinto, 1993). For example, family responsibilities can inhibit integration by limiting a student's contact with the social networks of the program (Tinto, 1993), while faculty mentors can enhance integration by helping students learn the informal rules of interaction (Girves and Wemmerus, 1988; Gerholm, 1990). Students who are not well integrated into their departmental communities and cultures have been more likely to leave graduate school for other reasons; for example, poorly integrated students are less likely to put up with financial hardship (Lovitts, 1996). Thus other factors implicated in attrition actually mask an underlying issue of integration: they have their

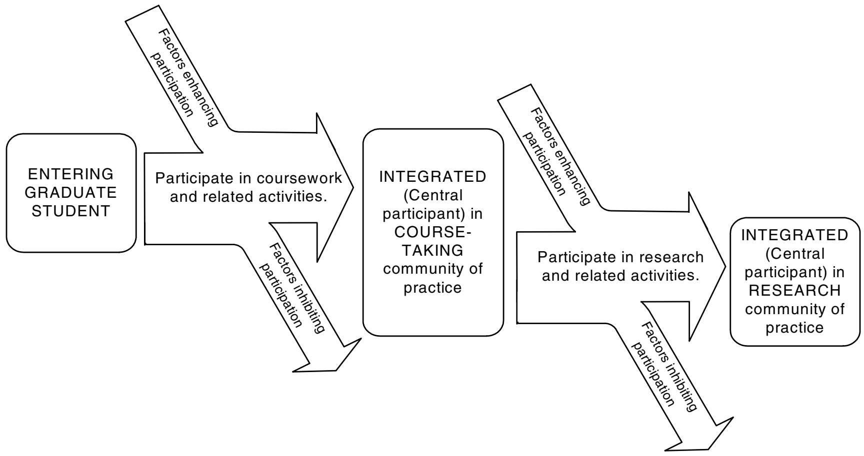

Figure 1. Doctoral students in mathematics need to participate in, and become integrated in, two distinct and sequential communities of practice.

effect through their impact on enhancing or inhibiting student integration to the communities of the department and program.

Combining these ideas of participation and integration leads a model in which experiences that enhance students' participation in the communities of the department lead to increased integration, which increases the likelihood of persistence, and experiences that inhibit their participation lead to decreased integration, which decreases the likelihood of persistence.

Mathematics doctoral students may actually encounter two distinct communities of practice in graduate school in the U.S. (Wiles, 1999), each of which has distinct "relations among persons, activity, and world" (Lave and Wenger, 1991) (see Figure 1). In the first three years (or so) of doctoral study in the U.S., students' primary activity is to participate in courses, leading up to qualifying exams. Once they complete these requirements, they can then begin looking for a research area and learn to do research. At this later stage, while the student's experiences are shaped by norms of the department and the broader discipline, those experiences closely reflect the student's interactions with a small number of faculty in the department (Tinto, 1993). Mathematics is a highly specialized discipline and many graduate students have few peers with whom to collaborate (National Research Council, 1992), so the communities of practice in which they participate at this later stage can be very small; this makes the relationship between the student and advisor crucial (Tinto, 1993). Thus the nature of a student's interaction with other department members is different in these two stages of doctoral study.

The rest of this paper describes a case study of one mathematics department, investigating the opportunities doctoral students have to become integrated into the communities of the department. Some previous studies interviewed current (Becker, 1990; Cooper, 2000; Hollenshead et al., 1994) or former (Stage and Maple, 1996) mathematics doctoral students about their experiences, and some have investigated faculty attitudes toward the teaching and learning of mathematics (Burton, 1999). I wanted to compare the experiences and perceptions of *all types of participants* in mathematics doctoral education – faculty and graduate students, continuing and departed students – *in the same department*, in order to understand the relationships between doctoral students and faculty as the students learn to become mathematicians. This paper focuses on the earlier stages of the graduate program – the first of the two communities of practice described above – in order to characterize it in more detail. Hopefully, once this model is refined, it will be useful in the future for studying more specific questions about attrition from and engagement with mathematics, particularly issues of gender, race, and class.

2. METHOD

This study was conducted between the fall of 1999 and the fall of 2000, in the Mathematics Department at a large, public research university in the United States. The focus of the program is on the Ph.D., with a Master's awarded along the way to the doctorate.

In the fall of 2000, there were 51 tenured and 6 tenure track faculty members in the Department (Department secretary, personal communication, September, 2001), and the Department enrolled 116 graduate students (103 full-time), of whom 74 (64%) were domestic. In the 1999–2000 academic year, 11 students received the Ph.D.; those students had been enrolled for between 4 and 12.5 years, with a median of 6.5 years (University website, 2001). Of the 199 graduate students who entered the graduate program between 1983 and 1987, 98 (49%) completed the Ph.D. and 3 were still enrolled ten years later (University website, 2001); that is, the attrition rate in this Department is about 50%.

This report is based on interviews that were part of a larger study, in which a total of 23 graduate students and 21 faculty members were interviewed. Of these, only the 18 graduate students in pure mathematics who were college-educated at U.S. institutions[1], and only 10 of the 11 pure mathematics faculty who were currently active in the graduate program are included here; the eleventh was excluded because of the failure of the tape recording during his interview.

Participants were initially recruited through email messages sent to the entire department (separately for students and faculty). In addition, particular individuals were invited to participate based on recommendations from other participants, or in the case of faculty, based on their roles in the graduate program. Participants were guaranteed confidentiality and the opportunity to review and comment on reports based on their interviews.

Faculty participants represented most major research areas in the Department, were all full professors who had been in the Department for most of their careers, and included some who teach first-year graduate courses. Interviews explored their 'mathematical autobiographies,' experiences as mathematicians, and roles as professors and advisors to graduate students (See the Appendix).

Graduate student participants had completed between one and more than six years of study at the time of their interviews, including 6 students who had left or were about to leave the doctoral program in mathematics, and 12 students who were planning to or about to complete their Ph.D.s; of those 12, six had previously considered leaving the program[2]. Nine of the 18 had completed all pre-dissertation requirements, and included students who were still taking courses to prepare for research and those who were completing the dissertation. Interviews explored their 'mathematical autobiographies,' experiences in undergraduate and graduate school, and decisions about continuing or leaving (See the Appendix).

Participants were ensured confidentiality, which several of them took special note of and explored with me in some detail before consenting to participate. In order to protect their identities, it is not possible to provide more specific demographic information. In particular, although both women and men were interviewed, as well as faculty and students from most major research areas in the Department, I cannot disclose their numbers without compromising confidentiality. Further, in order to avoid the assumptions and stereotypes that readers may bring to a reading of faculty and student experiences in mathematics, the gender of individual participants will not be disclosed; since most participants are male, masculine pronouns are used throughout.

The interviews were open conversations. In addition to asking for specific information and facts, Anderson and Jack (1991) call for the need to invite participants to discuss "the web of feelings, attitudes, and values that give meaning to activities and events" (p. 12) and to give them "the space and the permission to explore some of the deeper, more conflicted parts of their stories" (p. 13). These subjective and personal aspects of participants' stories were valuable sources of insight about their experiences within mathematics. However, encouraging participants to talk about these issues can be painful; some participants were discussing what they perceived to be failure (more than one confessed that they viewed their conversations with me as free therapy). To allow participants to avoid topics that were painful, while still leaving the interviews open to discussion of personal experiences and the meanings derived from them, participants were given an outline of discussion topics several days before their scheduled interviews, so that they had the opportunity to think about those topics, delete anything they did not wish to discuss, and add topics they considered relevant (after Burton, 1999). One participant declined to answer the question "How did you become interested in math?" claiming that he was not interested in discussing himself; no other deletions were made.

The outlines for graduate student and faculty interviews served as a starting point for the interviews, but each interview was different, covering those parts of participants' stories that they thought were relevant, and the questions each participant was asked differed according to their stories. Also, if interesting issues came up in one interview, then subsequent participants were asked about that issue. For example, one of the earlier participants spoke about difficulty of mathematics; since it seemed relevant to the students' and professors' attitudes toward teaching and learning, other participants were asked if they thought mathematics was more difficult than other fields.

Interviews took place in a private office on campus with one exception, a graduate student who had left a few months earlier was interviewed by telephone. The graduate student interviews ranged from 1.25 to 6.5 hours (median 2 hours), in up to 3 separate meetings. The faculty interviews were much shorter, ranging from 1 to just over 2 hours (median 1.6 hours). Interviews were audiotaped and transcribed.

Unless otherwise noted, ideas are described below that were discussed by at least two-thirds ('most' in what follows) of the participants (i.e. at least 7 out of 10 faculty or at least 12 out of 18 graduate students); 'some' or 'several' of the participants means fewer than this. As discussed earlier, the questions the participants were asked differed according to their stories, and as a result, not all participants talked about the same topics. If a topic was mentioned only by a small number of participants, this does not mean that the other participants disagreed. Such disagreements are described if they were voiced. Otherwise, small numbers suffice only as what mathematicians call an 'existence proof': they are evidence that at least some people think these things, but do not imply how many.

When it is useful to do so, quotes are included from both professors and graduate students, or from both continuing and departed students. In the text that follows, quotes are presented because they are particularly articulate or clear in expressing common themes discussed by the participants. Quotes were edited for readability, removing stutters and distracting expressions such as ‘uh’ and ‘you know,’ and references that might reveal participants’ identities were obscured. Graduate students are identified as either continuing (they were enrolled at the time of the interview and intended to complete the Ph.D.) or as departers (a departed student had already left the program without the Ph.D., and a departing student told me that he intended leave soon), and sometimes as advanced (he was pursuing dissertation research).

3. Results

The factors the graduate students and faculty discussed that affected student participation and integration in the department and program fell into four general categories: the beliefs and attitudes of the faculty toward learning and teaching, the program structure, learning in classes, and relationships with faculty outside of the classroom. In each of these categories, both students and faculty described either a lack of opportunities for participation, or explicit obstacles to participation. For the faculty, these obstacles were often intentional, in order to force students to prove themselves before important resources were invested in their further development. To the students, these obstacles were often frustrating, and interfered with their ability to learn mathematics.

3.1. *Faculty beliefs about learning and teaching*

Two interleaved assumptions are often made in popular discourse about mathematics: that mathematics is a very difficult field of study, and that only some people have the talent required to be successful at mathematics. This may explain why it is socially acceptable for someone to admit that they can’t do math: it’s just too difficult for the average person. Most of the faculty participants in this study agreed that mathematics is very difficult, some of them specifying that it is more difficult than other subjects.

> In mathematics you really have to do something which may in fact be impossible and you don’t know in advance whether it is or isn’t impossible and that’s psychologically intimidating. In that sense it’s hard. You also have to come up with really new ideas, or most of the time to prove a new theorem, it requires some new idea. I don’t really think that in most areas to write a Ph.D. thesis you need a really new idea. You have to do some new work that hasn’t been done before, but I don’t think you really need a new idea. That I think is harder. (faculty)

Two faculty members disagreed, one describing mathematics as ‘simple,’ because work can be confirmed as either correct or not, and the other calling it 'easy,' since all mathematics is based on a small number of ideas, while one professor said that mathematics is not necessarily harder, but it is different than other fields.

The faculty participants generally believed that graduate study in mathematics requires a lot of hard work. Some of the professors said that they thought that some graduate students are not willing, or are not able, to work as hard as is necessary to do mathematics.

While mathematics is often described as objective and emotion free, mathematicians have strong emotional reactions to their work; for example, when they describe the euphoria that comes from solving a problem (Burton, 1999). All but one of the faculty participants eloquently described their work in mathematics as exciting, and their appreciation for the beauty of mathematics. They freely used words like 'beauty,' 'pleasure,' 'delightful,' and 'pretty.' One professor's eyes filled with tears as he described the first course he took from the person who later became his advisor, saying, "Well, I almost cry when I think of it ... I never saw anything like that." These positive descriptions they give of their work in mathematics provides a stark contrast to the ways they spoke about the need to work hard in mathematics, particularly in graduate school, using words like 'perseverance,' 'persistence,' 'stamina,' 'tenacity,' and even 'pain.'

> The best frame of mind when you come here is to basically be prepared for pure hell for the first one or two years. (faculty)

The graduate students also used negative words to describe the work of graduate school, but from a different point of view: they used words like 'fear,' 'intimidation,' 'stress,' and 'hostility.'

> The qualifiers [qualifying exams] are such a source of fear, they're just such a source of stress because you really have to focus on them. They sort of detract from enjoying what you're doing. (continuing graduate student)

But most faculty participants agreed that hard work alone is not enough – success at mathematics requires talent. Some of the faculty described this talent using words like 'smart,' 'genius,' or 'insight.'

> You can give some hints or sometimes you could see a theorem and say well, it's clear that you should attack it this way because, but sometimes it's genius that tells you which way to go. That's why there are really smart mathematicians because there isn't an obvious approach to the subject. (faculty)

Two faculty members said that people who have mathematical talent are not necessarily smarter than others, but that they possess a particular type of talent.

> Why don't we have more people that can run a 4-minute mile? Genes are different. There is mathematical talent. You can train everybody to run a mile, 96% or something like that in 7 or 8 minutes.... But there is a level of abstraction in

mathematics that you get to eventually that many people are just not comfortable with... Some of them are much brighter than I am but there's just a basic spark and a way of thinking that some of them just don't have and they never get it. There are different ways of thinking and some people don't have the streak of curiosity, the ability to see little things and then big things. I'm perfectly willing to admit that that's true. I could never run a 4-minute mile. (faculty)

Four professors said that a student either has the ability or not, but that it cannot be learned or developed in graduate school. Some even explicitly described mathematical ability as innate.

It requires a particular cast of mind. I can't describe it. It's something like perfect pitch. There are people who have it and people who don't have it. (faculty)

Even while some of the faculty participants expressed doubts about whether students can be taught to do mathematics, only one of them expressed a belief consistent with the view that anyone can learn mathematics – up to a certain level. None was even convinced that all graduate students in the program could succeed at mathematics, even though these were the students that they had chosen to admit. Half of the faculty participants spoke of the difficulties in predicting who would succeed, and several spoke of weaknesses in the admissions process.

When I asked the graduate students what it takes to succeed in graduate school, some of them attributed their success to working hard, but few of them spoke of talent; four graduate students specifically denied that talent determined who would succeed and who would not. Instead, they thought that it required determination, focus, and for some, luck.

I wish I could say that it's some sort of internal attribute that people have, that it's persistence or playing by the rules or doing what people tell you to do, or brilliance or anything like that, but I don't think it's any of those things... I think it's primarily luck... Even brilliance is not enough because people sometimes pass their quals after a year and then they just stumble around and then they stumble away. (advanced continuing graduate student)

While the faculty spoke of the need for graduate students to work hard, and some described their concerns that the students don't work hard enough, some graduate students perceived that the faculty expected them to be single-mindedly devoted to mathematics. Most students described the pressure or guilt they experienced – from their advisors, other faculty, or even from other students – over having other commitments in their lives, and the difficulties of balancing a life outside of mathematics with the demands of the program. Some students who were close to completing their degrees said that they could have been done more quickly if they had been more focused on their work, but that it would not have been worth the price to them.

As we will see below, these beliefs on the part of the faculty provide an explicit obstacle to students' participation in the program, in that students are required to prove themselves *first*; only after they have proven themselves would they have opportunities to participate in meaningful ways in mathematical practice. This forms a sort of a 'catch-22,' since it ignores the way that meaningful participation might enhance students' abilities and skills at mathematics.

3.2. *The structure of the program*

The first few years of graduate study in this department, and in most U.S. mathematics departments, consists of students taking courses in preparation for the qualifying exams (National Research Council, 1992). Given the faculty participants' beliefs about talent, it is perhaps not surprising that they described the ways in which they use those courses and exams to predict who had the ability to succeed, and to 'weed out' those who did not have that ability. Three professors stated or implied that they were doing students a favor by helping them to learn whether they should invest the time in mathematics, and by saving them from 'wasting' time if they were unlikely to succeed.

Three professors described ways in which their colleagues looked to limit which students go on in the program. According to one professor, "They're in a hurry to kick people out of here." Courses and qualifying exams were described as opportunities for students to prove, or discover, whether they have the ability to succeed, and instruction was described as an opportunity to put students to the test. For example, two professors voiced sentiments similar to this one:

> I don't think the difficulty is that you don't have the knowledge. I think you don't know whether you even belong here because you haven't been tested... You have to have the talent and I think the reason that we have the dropout rates is because many American students maybe don't have the talent and don't know they don't have the talent because they've never been tested. They've never had a tough linear algebra course or a tough advanced calculus course... Without that you really don't know if you're a good mathematician. I think upper level undergraduate math should be a deciding point and in most schools it isn't. (faculty)

Some U.S. mathematics departments, under the twin pressures to admit more domestic students and to staff undergraduate courses with teaching assistants, have liberal admissions policies (National Research Council, 1992). Several students perceived this to be the case in this department, saying that students are admitted to the program who are unlikely to succeed, just to meet the department's need for teaching assistants, or to increase the number of American students. The more successful programs are those that provide the support necessary for students to make up for deficiencies in their preparation; however, in many programs this is not done (National Research Council, 1992), so that students who need something other than the standard program of study might not succeed. In the department studied here, some students whose undergraduate coursework was not adequate preparation for graduate courses offered still were admitted to the program, and were expected to enroll in the same program of study as better-prepared students. Although there is a mechanism for admitting students 'with deficiencies,' it has rarely been used (Department secretary, personal communication, January 8, 2002) and none of the participants in this study had had that option. Instead, students were required to take at least two doctoral-level courses each semester in their first year (Department website, 2001). Students whose undergraduate coursework left them feeling unprepared for the introductory doctoral courses sometimes might have wanted to take some undergraduate or masters-level courses in preparation, but they were usually advised not to take those lower-level courses. According to one professor,

> I do think it's very important that they're able to take [doctoral]-level math right away when they come here because I think this is the only way we're going to quickly be able to predict whether or not they're going to make it. It's been our experience that if people take only [masters]-level courses the first year, they're not going to make it. The attrition rate is enormous for them. Basically it's a waste of their time and a waste of ours. (faculty)

These students had little opportunity to fill in the gaps between their undergraduate courses and the courses they take in graduate school. About half the students described the inadequate preparation they had for their first-year courses. Some of them felt that the courses were designed so that the only students who really learned from them were those who had covered the material before. In particular, the graduate student participants (all of whom were college-educated at U.S. institutions) felt that they were at a disadvantage compared to students educated outside the U.S., whose undergraduate training in mathematics was at a much higher level than their own. This student described having been very successful as an undergraduate, but he had difficulty in some of his graduate courses (although he received good grades).

> There doesn't seem to be a very good bridge between the undergraduate material and the graduate material. You come in, you take [doctoral]-level courses, they tell you to do it even if you're not sure you should. Maybe it would be good for a lot of us to take some undergraduate courses first, especially those of us who came from smaller schools and didn't get as rigorous a background, but it's just . . . not mentioned. You come, you're hard core, you do it, and if you don't [snaps], too bad. No one cares. You're just a failure, you're just one of those people who couldn't cut it. . . [Faculty name] was talking once about how, "Yeah I think half

of our attrition problem is these people from small schools just can't cut it."
(departed graduate student)

The qualifying exams served as another mechanism for weeding out students who do not possess the talent to do mathematics. In particular, the faculty described the qualifying exams as having two main purposes: to serve as a filter, helping students understand if they can cut it or not, and to force them to absorb and synthesize a large amount of information, certifying that they have breadth of knowledge in mathematics.

Some students recognized these goals as well, and several of them described how much they had learned through the work of preparing for the exams. However, half of the students felt that the qualifying exams were unreasonable measures. The problems they described were that the courses did not adequately prepare them for the exams, despite the description in program documents that the courses were designed to do just that (Department website); the exams were inconsistent in difficulty, both among the five areas in which they were offered and across time; that they were too difficult, especially since the deadlines for completing them had been made shorter and stricter; that they were demoralizing and intimidating; that they tested for obscure 'tricks' rather than for important concepts; and that the grading was arbitrary. This continuing graduate student passed both qualifying exams early in the program, both on the first try:

I don't think they have anything to do with whether or not you'll be a good researcher or even an effective one. They weed out people, they intimidate people, they demoralize people, they give a few people a great psychological benefit which I think could be achieved in other ways.
(advanced continuing graduate student)

Four faculty participants also described some limitations of the qualifying exams. They described ways in which they are an unreasonable measure of ability to do research, or that they test for the wrong things.

Doing mathematics isn't exactly like answering qualifying questions... When I do math I try to keep in my head a lot of questions, most of which I can't solve and then my success is measured by the ones I've solved. It's measured by the papers I've written, and not the ones I've failed to write, and so I think this is a little bit artificial. It does demonstrate some overall competence... It is sometimes used as a screen to decide who is capable of writing a thesis. I think that's probably wrong. It may screen people but for the wrong reason. (faculty)

Overall, rather than defining instruction as an avenue to teach students to be mathematicians, these faculty described instruction as an avenue for students to prove themselves. This approach removes most of the responsibility for teaching and learning from the faculty, and instead places that responsibility almost entirely in the hands of the students, requiring students to develop a certain degree of competence *before* they can interact with faculty in meaningful ways. This program was structured to actively limit – rather than enhance – students' participation both in the program and in mathematical activity.

3.3. *Participation in coursework*

Although many of the faculty participants devoted substantial time and effort to their teaching, most of them described their goals for teaching graduate courses in terms of communicating the main ideas of the subject, rather than sharing their excitement or vision about mathematics. They primarily lectured, with a focus on teaching as telling students what they need to know.

> In each course I might say this is the key idea and I want to get that across . . . The requirements of the teaching and the requirements of the subject are the same, namely make it as clear as you can and express it as cleanly and as beautifully as you can. (faculty)

Despite the mathematicians' descriptions of the pleasure of doing mathematics, only two of them spontaneously spoke of their attempts to communicate this enjoyment to their students.

> With the young people, I would say go be enthusiastic, that's the main thing. Love it. (faculty)

Although the graduate students complimented some lecturers for their clarity or for making the material interesting, these compliments were rare. More frequent were their complaints about the teaching of the courses, including the lack of interaction between the instructor and the students, difficulty discerning the important information, incomprehensible lectures, non-English textbooks, and the lack of motivation or connections among mathematical ideas and the mathematical 'big picture.'

> He'd come in and he would race through the stuff on the board and we would furiously copy down what he was doing and it seemed like just streams and streams of words, signifying nothing. Then we'd have a month of this with no homework, really no indication of what on earth was going on, not much in terms of why we were doing what we were doing or where this was going. Just, "here's a lemma," "here's the proof," "here's another lemma," "here's the proof," "here's a theorem." Very little motivation, and I think I didn't see the whole big picture. (advanced continuing graduate student)

Most graduate student participants complained about the lack of feedback mechanisms in their courses. When work was assigned in class, students said that it facilitated their understanding of the course material, helped them keep up with the course, allowed them see if they could do what was being required of them, prepared them for the qualifying exams, and gave them an opportunity to interact with other students over the material while working on the assignments.

> Maybe I just got stuck with bad teachers but having classes where there were no tests, no quizzes, no sample problems, no homework, no exercises, no books, that there was just nothing for me to learn from... He wouldn't give us sample problems, like sample homework, and people were asking him for sample homework problems, not even things that he would collect, just sort of this is what we're doing in this class. These are things you should be able to handle right now.
> (departed graduate student)

When work was collected and students were given feedback on it, they said that it helped the professors know how the students were doing and further helped them learn, through the feedback they received on their reasoning and writing. However, in many of the first-year courses in this program, professors did not give students feedback on their work; in some classes, work was not even collected or graded.

> The first homework assignment I had gotten back and it was 8 out of 10 or something like that... The second homework assignment I got back, two-thirds of the way through and I got 1 out of 10 and the comment was, on one of them, "you didn't understand the problem," and on another one there was just an X, and that was when I decided that this is my last day in this course because I have no idea what I'm doing, why I'm doing it wrong, what I'm doing right, there's no feedback of any kind. (continuing advanced graduate student)

Half of the faculty participants acknowledged the importance of grading to student learning; this professor stated it most strongly:

> Absolutely all of us know that if we wanted to improve teaching... we should grade a lot more. We should not have graders. We should read the papers of the students.... [P]eople need to practice and to be told that what they are doing is just not satisfactory, and try next time to do better. You are not going to get students to write well without asking them to write in the first place, without checking that they write well.... I'm not saying that grading is a pleasant thing but it's a necessary step in learning. (faculty)

When asked if he grades papers of his own students, he said, "Well, I am not a saint, you know."

Another professor described how difficult it is to grade mathematics.

> It's very hard to say exactly what's wrong, especially if the student says true things but the logic isn't right. How do you criticize a proof where the student writes five true things but you don't see the reasoning when you're going from step one to step two? It's hard to criticize that. It takes a lot of skill and something I usually fail at.... I don't like the grading, I find that very painful. (faculty)

Four graduate students expressed the view that the role of the qualifying exams in weeding people out and in ensuring that they learn the mathematics would be achieved more effectively by providing feedback on student work in classes, and by making the grading in courses more reflective of student understanding.

> I think part of the reason qualifying exams are there is so that teachers don't have to worry too hard about the grades they give, and I think a lot of the professors here are more interested in research than in accurately grading their students, and that's too bad. In the best of all possible worlds, I would get rid of qualifying exams and make it grade-based in the classes and make teachers give homework every week so that students actually were able to follow the course. (continuing graduate student)

A few students said that they felt comfortable asking questions in or out of class, or complimented particular professors for how well they responded to questions. However, half of the graduate students – both continuing and departed – described that they either could not ask questions, or felt that they were chastised when they did.

> In the beginning I went to professors' offices very often and I was received so coldly and my questions were really trivialized and I was sort of berated for even asking them that it just became such an unpleasant and unproductive experience for me that I just stopped going. (departing graduate student)

Rather than the coursework building students' enthusiasm for mathematics and involving them in authentic mathematical work, coursework distanced students from mathematics, making it more difficult for them to learn what they needed to learn to participate effectively in mathematical practice.

3.4. *Relationships with faculty outside the classroom*

Tinto (1993) talks about three stages of persistence toward the doctoral degree. In the first stage, students adjust to the academic and social communities within graduate school and they make judgments about the relevance of the program to their career goals and the desirability of membership in the community. In the second stage, students develop knowledge and skills – or 'competence' – deemed necessary for doctoral research, culminating in comprehensive exams. The third stage of persistence is the completion of a dissertation. According to Tinto, student persistence through the first two stages (the first of the two communities of practice described earlier) reflects not only individual characteristics, but also interactions between students and faculty in the department and program. These interactions play a role in developing competence and also affect the judgments others make about those competencies; faculty judgments of student competence within the classroom are shaped also by social judgments arising from interactions outside of the classroom. Thus interactions between students and faculty outside the classroom are crucial.

In this department, almost all of the students interviewed (15 out of 18) described the limited or negative relationships they had with faculty, and for some students these topics formed a relatively large part of their interviews. While students described different aspects of these relationships, there were several common themes among their comments, which are the subject of this section. In contrast, the faculty were largely silent about their interactions with students at the earlier stage of the program, perhaps because they had so few such interactions; while their comments are presented here when relevant, they spoke about these issues only rarely.

Eight of the students reported that they felt 'invisible' or ignored. A number of students, including both first-year and advanced students, felt that first-year students needed more guidance and interaction with faculty. Many students felt overwhelmed during their first year, from the combination of the demands of coursework and for some, the adjustment to being in such a large department.

> I think a lot of people have a similar experience to my first year where I was thrown in with not that much guidance, not really understanding my classes, falling behind and not knowing how to get help. I wasn't real comfortable going to my professors. I think it's better when the professors of the first year classes actually reach out and talk to the students and I don't think that always happens. Sometimes they just come in, they talk and they leave, and the students are left with not knowing what to do. (advanced continuing graduate student)

In particular, eight of the graduate students said that the faculty did not care about the graduate students. Some graduate students reported that faculty did not bother to learn their names.

> It bothers me that there are more secretaries that know my name than professors . . . I was trying to think of how many professors would know my name, and it's pretty minimal.. . . Even [name] who I really liked, I had him for 2 semesters, including a [topic] class where there were only 8 people in the class, and I wasn't a bad student by any means, and he didn't know my name after two semesters.. . . When I told her I was dropping out, my mom asked me if any of my professors had tried to change my mind about that and it kind of made me laugh because the number of professors I would have that even know my name is so minuscule, the fact that any of them would try and change my mind is just ridiculous.
> (departing graduate student)

Some students believe that the professors do not pay attention to graduate students until they prove themselves by passing their qualifying exams, a view which is consistent with the faculty's requirements that the students prove themselves.

> They admit you into the program and then you have to prove yourself by taking the quals and they don't give you a lot of attention until you've taken the quals. I'm assuming then that once you've taken the quals then they start giving you more attention. (departing graduate student)

> Not only does it say to me, "Ok, I got past a hurdle," but I've noticed other professors have congratulated me or will start to notice me a little more. I feel like somehow the department thinks I'm making progress, whether I actually am or not, and whether I'm actually smarter than my friends are or not. Somehow I've made a check in the checklist and that makes a big difference to feel like somehow you were considered to be good enough. (continuing graduate student)

One graduate student, who was finishing his dissertation at the time of the interview, described his frustration that he did not get any more attention after passing his exams than he did before.

> I had the impression early on that the faculty weren't paying all that much attention to me and I was assuming that it was sort of well, I haven't proved myself yet. I haven't passed my quals, I don't have an advisor. When I do those things that will change and it didn't, which is still a source of frustration to me.
> (continuing advanced graduate student)

Two advanced students described how accustomed they had become to this lack of attention. Later in their studies, when someone did show an interest in them or their work, they were surprised.

> I've very, very rarely had faculty members say, "So what are you working on? Tell me about your work." And when it has happened, it's happened in the last year or so.. . . After I gave a talk on Thursday one of the [faculty] who had been at the talk came up to me afterwards and in the context of talking about the talk, said "I've worked on some somewhat related things. Maybe we should talk sometime.". . . That is a novel feeling to me and it shouldn't be, especially not when I've been around as long as I have. (advanced continuing graduate student)

So, even after students 'prove' themselves by passing qualifying exams and making progress through the program in other ways, it is still difficult for them to build meaningful relationships with faculty.

Several graduate students felt that the professors were unfriendly, uncaring, or even hostile.

> When I first got here, the graduate advisor was . . . a very unfriendly person. I talked with him a couple of times and then I'd see him in the halls and he would just look away. I hated that. (advanced continuing graduate student)

> Of all of the things that I dislike about the profession of mathematics have to do with just the fact that sometimes individuals decide to be unpleasant about this thing or that thing and it's not got anything to do with mathematics. If I had to change the profession of mathematics, I would make people nicer. And I think that that would have lots of repercussions.
> (advanced continuing graduate student)

Among the graduate student participants in this study, two said that they felt like they were part of a community, and two others remarked that they enjoyed the annual picnics that are customary in their research area and the opportunity they provided for informal interactions between the students and faculty members. However, overall, the predominant interactions between students and faculty in this department were in the classroom, when the professors stood in front of the room to lecture. Seven students said they wanted more interaction with the faculty outside of class; several advanced students noted that they did not have any meaningful relationships with faculty. Five of the six graduate student participants who were close to finishing their dissertations noted that they had not had any mentors in graduate school.

> Mentoring would be sort of an advanced version of collaborative work and just feeling collegial and feeling like we're doing the same sort of thing and cooperating and talking about the same stuff, and there isn't even that basic level so there certainly isn't any mentoring going on. (advanced continuing graduate student)

The moral support and encouragement provided to graduate students by mentors have been found to be particularly valuable (Cooper, 2000; Hollenshead et al., 1994). The few graduate student participants in this study who did talk about receiving such support said that it made a big difference to them.

> I think the main reason that I'm still here is having gotten support from him and it's really sort of more emotional support and moral support at the one critical time.... The time I thought most about leaving, my advisor was there and basically said, "No, you shouldn't do this. You are close to finishing. I know, I do believe you can do this." (advanced continuing graduate student)

However, such reports were the exception rather than the rule. In this mathematics department, each student is assigned an initial advisor at the beginning of his first year who usually remains in that role until the student passes qualifying exams and finds a research advisor. Many students interviewed (11 out of 18) found this system ineffective, and rarely spoke with or got useful advice from their advisors; some did not even know who their advisors were. As a result, they did not receive the advising they felt they needed or expected. Half of the graduate students said that they suffered in important ways from the lack of advising, which might have helped them make better decisions about courses to take and could have given them a more clear idea of what to expect.

Although most of the faculty participants in this department spoke positively and sometimes fondly of their advisors in graduate school, most had had very distant relationships with their advisors, with little interaction. Several said that they had spoken with their advisors only three or four times in their entire graduate careers. Only one faculty member bemoaned the poor state of advising in the Department.

Eight of the students described wanting to know more about how their professors think about mathematics, how they approach solving problems, or how their work fits into the broader mathematical landscape.

> I've never had a conversation of that kind of any depth with any of the professors at this place so I really have no idea of what anyone here feels. . . . Perhaps I will get that later in my education but right now I perceive they're just spewing material at me so. . . I can't perceive the feelings behind it.
>
> (continuing graduate student)

Three students I interviewed – one continuing and two departing – said they came to graduate school expecting to be treated like junior colleagues, and were disappointed that that was not what they found.

> I built up an expectation that in a graduate program you have a community of mathematicians and mathematicians in training. They view you as junior colleagues that they want to mentor, and as a result of that, they're going to create an atmosphere that's going to be conducive to you becoming a mathematician. . . . I just have not found that to be really the case here and that was really one of the most bitter disappointments of all. (departing graduate student)

One benefit of being treated as a junior colleague is that it can help a student develop 'tacit knowledge,' the unspoken cultural rules and informal knowledge of the discipline that graduate students need to master, and which they develop through contact with more experienced researchers (Gerholm, 1990). If students have limited interactions with faculty, then they will have limited opportunities to develop tacit knowledge about the discipline. Several graduate students noted that there were few opportunities to develop this type of knowledge, since there was so little interaction between students and faculty, although a few students described one professor who routinely takes his graduate students to conferences and does other things with them to help introduce them to the discipline more broadly. Many students spoke of how little they could understand in colloquia and seminars; two students had organized student seminars in which material was presented at a level more appropriate for graduate students. Two advanced graduate students enjoyed support they received from one particular new faculty member, who helped them develop some tacit knowledge about mathematics.

> He's telling all the graduate students, "Yes you should go to the colloquium." And then. . . a couple days after he runs a session where he goes over some of the concepts that were discussed there.. . . I was also thinking that, "Boy I wish someone had been doing this 5 years ago." I would have gotten so much out of this. Another thing he's doing in conjunction with that is what he's referring to as a 'tricks of the trade' workshop, things that you have to know as a working professional mathematician that you never really get taught explicitly but somehow you need to know. (advanced continuing graduate student)

Lave and Wenger (1991) claim that learners need 'access to peripherality' in addition to legitimate participation, and point to "the crucial character of broad, and broadly legitimate, peripheral participation in a community of practice as central for increasing understanding and identity" (p. 85). In order for students to become central participants in the practice of mathematicians, they need access to all the means of membership in that community. "The important point concerning learning is one of access to practice as resource for learning, rather than to instruction" (Lave and Wenger, 1991, p. 85). 'Old-timers' set the stage for the activity of the newcomers. The activity of the community provides a 'curriculum' for those students who have legitimate access to that activity; that is, students learn through their participation in the activity of the academic community. "Participation in the cultural practice in which any knowledge exists is an epistemological principle of learning. The social structure of this practice, its power relations, and its conditions for legitimacy define possibilities for learning (i.e. for legitimate peripheral participation)" (Lave and Wenger, 1991, p. 98). That means that graduate students need to interact with the 'old timers' in order to learn to participate in the practices of the mathematical community. Without those opportunities for participation, students' abilities to become integrated in the communities of the program are severely limited. In the next section, I will argue that those students who were able to overcome that obstacle were those who arrive at graduate school with particular types of cultural capital; that is, they had had experiences outside of graduate school that helped them become integrated to the mathematical community.

3.5. *Deciding to leave or stay*

Among the graduate student participants, there were six people who had left the program or were about to leave, and six continuing students who had considered quitting but decided to continue in the program. What explains the differences between the outcomes for these two groups of students?

Some degree of doubt, and some students leaving, might very well be normal and healthy. For example, one graduate student considered leaving in his first semester when the stress of balancing teaching and coursework seemed overwhelming, but he decided to work carefully on managing his time; this improved his situation, so he decided to stay. Another student was disappointed not to have had the opportunity to study a specific topic area that interested him; he said that he didn't have the motivation to study for his second qualifying exam in an unrelated area, so he decided to leave.

Three students had doubts about epistemological issues. Sometime during his dissertation research, this student began to question the meaning and value of his work in mathematics.

> I still am very happy with the department life here but my disillusionment and reservations I've had at times had more to do with the subject itself. . . The kinds of questions that people are proving theorems about these days are really very hard to motivate why anybody should ever care.
>
> (advanced continuing graduate student)

Much to his surprise, when he asked his advisor about his progress, his advisor told him that he would be ready to graduate soon. He decided to stay: "If I've got enough to write up as a thesis, I ought to graduate." However, another student who had similar concerns about the value of his work in mathematics came to those doubts earlier in his program, shortly after finishing his qualifying exams, and had no similar incentive and support to stay in the program and eventually left. A third student who was having doubts was concerned about the difficulty of making connections among big mathematical ideas, given the focus on detail in each of his classes, saying, "I didn't realize just how huge the landscape is. I had no idea." While he was planning to continue in the program at the time of the interview, he left the program a year later.

Aside from these personal and epistemological concerns, there are some interesting patterns, which support the idea that student *participation* in mathematics and their *integration* into the mathematical community are important to their success.

Participants were asked about their mathematical autobiographies, including how they had become interested in mathematics and how they came to be where they are now. For most of the faculty and many graduate students, their interest in mathematics developed very early, some of them telling stories that went back to their pre-school years. Half of each group said that they liked mathematics because they recognized that they were good at it.

> According to my parents it started when I was probably three or so.. . . I was good at it, I liked it, it was very pretty. (faculty)

Four of the six departing students said that, even though they were always good at mathematics, they had had diverse interests when they were young and did not develop a commitment to mathematics until late in high school or in college. An additional departing student had been intensely interested in mathematics beginning in grade school, but made an intentional decision sometime in high school to broaden his interests and to be less focused on mathematics. In contrast, nine of the 12 continuing students were focused on their interest in mathematics no later than the eighth grade. Equal proportions of departing and continuing students were involved in extracurricular math activities in high school (e.g. competitions or clubs), but the differences between these groups are in their *focus* on mathematics at that age. Four of the departers felt that they could not succeed in graduate school because they had not been as absorbed in mathematics from as young an age as some of the other graduate students.

> It's not just being an intelligent person or having the tools in your brain. I think it's pretty obvious that I had that from a young age. But I think it's also that you have to study math, you can't take time out to do a lot of other stuff. You just have to be on that path from an early age. (departed graduate student)

Disciplines in which it is difficult to develop the tacit knowledge favor particular types of students (Gerholm, 1990).

> The combination of disciplines with much tacit knowledge and few possibilities in graduate education to acquire it would seem to favor students endowed with large amounts of . . . 'cultural capital,' i.e. a stock of knowledge, a frame of reference and a capacity to make the proper judgments called 'taste.'
> (Gerholm, 1990, p. 269)

Given the limited opportunities that students may have acquiring the tacit knowledge of the discipline of mathematics in this department, students who entered graduate school with a greater exposure to mathematics, who had spent more time exploring the discipline, may have been advantaged over those students who were newer to the field.

Three of the departed students reported having difficulty with coursework or with qualifying exams; indeed, it is possible that they would not have had the option to stay in the program. However, each of them told stories about ways in which they had sought advice and mentoring from their professors – they asked for help in learning – and said that that support might have made a difference to them. For example, one of these students felt berated by a professor when he struggled in a first-year class, and longed for the type of mentoring he had received in his undergraduate department (some of the continuing students also reported having missed the interaction they had had with faculty as undergraduates). His request to another professor for advice on preparing for the qualifying exam was rejected. In contrast, one of the continuing students reported that during his struggles to pass his qualifying exams, he had considered quitting, but the support and encouragement he got from a professor was a very important part of his determination to continue.

> When I went to talk to him, and just over and over he said, "I know you can do this. You haven't passed it yet but I believe you can," and it helped me to have an actual [mathematical area specialist] say these things to me.
> (continuing graduate student)

With that support, he believes, he eventually passed the exam. Similarly, another advanced student had doubts about continuing as he was beginning his research. The encouragement from his advisor made a big difference in his decision to persist.

The sixth departed student initially came to graduate school because he wanted to learn more mathematics; he said that he found the program dissatisfying. He wanted more interaction with his professors, but felt that they were not interested in his education.

Several students observed that the graduate student cohorts that formed a more cohesive support network had lower rates of attrition.

> I think looking back at the ranks of graduate students who have gone through our program; the ones who have done well are the ones who have found their niche in the community. And most frequently that's a community of people who are the same year. . . It's an interesting question to me to what degree the department can facilitate doing things like that. (advanced continuing graduate student)

Of course, each student's story is personal and unique. Despite this, most of the students – both departing and continuing – had similar descriptions of their relationships with faculty. They largely spoke with one voice about the ways in which they were left on their own early in the program, with little in the way of mentoring, teaching, feedback, advising, and moral support. What differentiated the students who had had doubts but persisted from those who left were two things. Some students who had doubts later in their programs had already developed relationships with some faculty member, who offered individual words of encouragement that made a difference; students who were earlier in their programs often did not have that form of support. For most of the remaining students, the continuing students had other paths to becoming integrated in the mathematical community – such as family members who were involved in mathematics, participation in research experiences as undergraduates, or involvement in mathematics from a very young age – that were independent of what happened within the Department itself. That is, the continuing students were those who arrived at the Department already in possession of cultural tools that were important to their success.

4. DISCUSSION

In doctoral study, intellectual and social integration are interrelated (Tinto, 1993). While intellectual integration involves sharing values with the community into which a student is integrated, social interactions within the program, both with other students and with faculty, are important parts of membership in academic communities. "Social membership within one's program becomes part and parcel of academic membership, and social interaction with one's peers and faculty becomes closely linked not only to one's intellectual development, but also to the development of important skills required for doctoral completion" (Tinto, 1993, p. 232). That is, a doctoral student needs to do more than just learn the content of the mathematics taught in classes; he needs to learn to participate in social and cultural practices. Students participate in those practices through interactions with faculty, both in and out of the classroom (Girves and Wemmerus, 1988; Lave and Wenger, 1991) which then makes it more likely that they will persist to complete the Ph.D. (Lovitts, 1996; Tinto, 1993).

Students who are treated as 'junior colleagues' have been found to be more likely to stay in graduate school and complete degrees (Berg and Ferber, 1983; Girves and Wemmerus, 1988; Nerad and Cerny, 1993). Berg and Ferber (1983) reported that graduate students who earned a doctorate (compared with those who enrolled in doctoral programs but did not earn a doctorate) were 3.4 times as likely (based on an odds ratio) to have reported being treated as a junior colleague by at least one male faculty member, and 4.8 times as likely to have come to know two ore more male faculty members quite well. Conversely, students who feel they are treated as 'adolescents' are less likely to complete degrees (Nerad and Cerny, 1993). Infrequent interactions between students and faculty, and lack of both faculty mentoring and departmental advising have also been associated with high attrition or long time to degree (Nerad and Cerny, 1993).

In this Department, students' interactions with faculty members during the first several years of graduate study mostly occurred while they listened to lectures in classes; even within the classes, there was often little interaction, either orally or in writing. The graduate students perceived what the faculty described: that the faculty were not interested in investing in the students until after they had proven themselves. The students described the limited and often negative relationships they had with faculty, and an almost complete absence of mentoring or advising.

Programs characterized by sporadic evaluations have also been associated with high attrition and long time to degree (Nerad and Cerny, 1993). The absence of graded assignments in most of their classes left students with few opportunities for feedback on their mathematical thinking. Several described their frustration from not being able to get their questions answered. For many of them, the first meaningful feedback they get about their work was on the qualifying exams. Although students can ask to review their exams after they are graded, the feedback that the exams provide is generally only in the form of the grade, and the 'curve' that determines whether that grade is a passing or a failing one. Given that the exams are offered twice a year, and that students have a limited time in which to complete the exams, the feedback they receive from the qualifying exams is unlikely to be useful for their learning in any substantial way.

The primary social arrangements within the Department do not facilitate interaction between students and faculty; instead, they isolate students so that instead of interacting with faculty, most of the graduate students' interactions were with other graduate students, both socially and as they worked together on coursework and preparing for qualifying exams. This is consistent with Wiles' (1999) conclusion that graduate students engage in a distinct 'course-taking' community of practice in the first few years of graduate study.

In constructing and maintaining these social arrangements, the faculty might be emulating their own experiences as mathematics students and as mathematicians. Mathematicians have been characterized as being very independent (Fennema and Peterson, 1985), as many mathematicians' biographies will support (Fennema and Peterson, 1985; Helson, 1980; Henrion, 1997). This is not to say that mathematicians do not collaborate; they do, as other authors have written about (Burton, 1999), and as the participants in this study described. However, when I asked these mathematicians about the *nature* of their collaborations, they described partnerships in which most of the work was done independently and individually by the collaborators, and put together periodically; some had even written papers with people they had never met. The distant relationships the faculty had reported with their own advisors and the independence with which they do their work might represent the type of 'mentoring' relationship that the professors emulate, particularly with students who are not yet ready to begin research. They may assume that their students are as independent as they were, and therefore do not offer their students any more contact than they needed themselves. It is also possible that they might not be aware that other arrangements are possible, since they may have no direct experience in a more connected intellectual relationship. Of course, in considering the fate of the students, this is a 'chicken-and-egg' question: are mathematicians independent because that is a style that is best suited to the nature of mathematics, or, since the social structure is one that requires independence, is it only the independent learners who succeed in mathematics? It may be that neither of these is the case, since the students who persisted tended to be those who had other avenues for participating in the mathematics community; rather than being as independent as they may seem, they may merely have been working independently of their professors but with other sorts of relationships and collaborations.

Causal attribution theory characterizes students' attributions for their successes and failures according to their locus of control (internal or external to the student) and stability (Bar-Tal, 1978; Fennema, 1985). Students' attributions of their past successes and failures have been shown to be related to their persistence at completing tasks (Bar-Tal, 1978; Dweck and Goetz, 1978) and in mathematics course-taking (Fennema, 1985). If a student attributes his failures to effort (an unstable, internal cause) and his successes to ability (a stable, internal cause) he is more likely to see success to be within his control, and consequently will be more likely to persist. If a student believes that his failures are caused by ability and his successes are caused by luck (an unstable, external cause) or by the nature of the environment (a stable, external cause), he will be more likely to see success as being beyond his control and he will be less likely to persist.

Although persistence in graduate school is far removed from the contexts of these studies about attributions, this theory provides an interesting model for the findings of the present study. Most of the faculty interviewed attributed students' successes or failures to ability and effort; that is, they described the causes of student success and failure to lie within the students. This removes responsibility for the success of their students from the faculty, which is reflected in the limited attention they pay to the students until the students prove that they are indeed likely to succeed on their own.

Burton's (1999) conversations with mathematicians indicated how little they thought about their teaching and "especially how little they attempted to convey the struggle and the pleasure which they had described to me of doing mathematics" (p. 140). In contrast, many of the faculty participants in this study were quite dedicated to their teaching. However, like Burton's mathematicians, "My participants, as teachers, are not exploiting their experiences. Nor are most of them giving learners a sense of the fun, excitement, challenge which holds them in the discipline" (p. 139). The work that they put into their teaching may have been more about the subject matter – the mathematics for which they have so much enthusiasm – than it was about the students who were the objects of the teaching.

On the other hand, the students attributed their successes and the successes of others to hard work and, for some, luck, but they attributed their failures and their struggles to features of the environment. Most of the students attributed their success to the unstable cause of effort, and described the pressures they felt to work harder and to minimize their commitments outside of mathematics. Several of them specifically denied that ability determined who would succeed. For some students, this attribution of failure to external causes led them to conclude that they did not have the ability to do mathematics. This is a particularly interesting juxtaposition to the students' statements that they initially liked mathematics because they were 'good at it,' and their descriptions of the satisfaction that comes from solving a problem. When suddenly faced with what they perceive to be evidence that they are not good at it, and they can no longer experience the satisfaction of the work, they may no longer be motivated to continue. The prediction from attribution theory that students' tendency to persist is decreased when they are faced with failure that they perceive to be beyond their control may explain the high attrition rate of doctoral students in this department.

5. Summary

Lave and Wenger (1991) argue that students' work is always peripheral to some community of practice, and that it is through that peripheral participation that students come to learn the practices of that community. As an example, they discuss high school physics students, whose work is not peripheral to the practice of physicists, nor are they on a trajectory to reproduce physicists. Instead, they are part of a cycle of social reproduction in which they are participating in a peripheral way to the community of educated adults. Similarly, doctoral students in mathematics, at least initially, do not engage in activity that is peripheral to the work of mathematicians. "To the extent that the community of practice routinely sequesters newcomers these newcomers are prevented from peripheral participation" (Lave and Wenger, 1991, p. 104), which then limits their ability to learn to be mathematicians.

Of course, this was a case study of only one mathematics department at one U.S. institution. Although it seems reasonable to conclude that students in this department would benefit from enhanced interactions with faculty and other opportunities to become integrated to the communities of the department, it is not clear to what extent this represents the U.S. mathematics community more broadly. However, a report prepared by the Mathematical Sciences Board of the National Research Council implies that this is not just a local phenomenon.

> For people who view the profession as a kind of priesthood, it is appealing to reduce numbers by keeping out all but the most worthy. However, there might be several negative consequences to such an approach. First, there would be the terrible human waste of labeling a large group of our most talented people as failures and choking them out.. . . Second, while Darwinian selection appeals to many mathematicians as a fair way to choose who succeeds, the playing field is often not as level as many would like to believe. In many cases, it's as though someone taught some of the animals how to use weapons and then accepted the

outcome of which animals survived as having been dictated by nature.
(Douglas, 1997, p. 43)

There are several approaches an academic community can choose to train its new members. Graduate students can be actively cultivated and provided with a nurturing environment that allows the largest possible number of them to flourish. Or, they can be put in a demanding environment with little support, so that those who are not already adapted to that environment will not survive. In reporting on this mathematics department, I told a story of faculty who believed that success in graduate school requires talent and hard work, but they did not describe what they did, or could do, as teachers to facilitate that success. Instead of viewing instruction as an opportunity to help cultivate the students into mathematicians, the faculty view the early years of the program as a time to weed out the students who do not have what it takes to succeed.

Many of the graduate students educated at this elite institution will go on to become faculty at colleges and universities throughout the U.S. and elsewhere, where they will be responsible for educating the vast numbers of students who will become teachers at the pre-college level. Etzkowitz et al. (1992) describe graduate research as a tradition in which "master scientists create successors in their own image as a form of asexual reproduction" (p. 159). Indeed, since teachers' own classroom experiences shape their beliefs and knowledge about mathematics teaching and learning (Fennema and Franke, 1992; Thompson, 1992), these graduate students' educational experiences are likely to pass on to other students in a type of 'domino effect.' That is, if the survivors of this educational environment teach the way *they* were taught, and the pre-service teachers they teach later teach the way *they* were taught, then it is critical, and alarming, to consider the effect of this model of graduate education on children learning mathematics in schools.

These issues of interaction, participation, and integration should be investigated among other members of the mathematics community, including students and teachers at other levels of mathematics education at a variety of types of institutions. Comparisons with other disciplines will be important to understanding the opportunities for participation that are particular to mathematics, and those that are more general characteristics of universities or of graduate study. And, since the disciplinary culture is caught up with broader social cultures, international comparisons will further illuminate how the ways that students participate in the culture of their departments affect their integration, and ultimately their persistence and attrition decisions.

Hopefully this model, in which participation in the academic and social communities of the department is a critical factor in doctoral student persistence, will help us understand why there are so few women and people of color in higher levels of mathematics in the U.S. (Herzig, 2002). It may be that it is difficult for people of underrepresented groups to participate in a disciplinary and departmental culture that was formed and has historically been populated by a much narrower demographic group. In order to cultivate a more diverse population of mathematicians, the mathematics community needs to examine its participatory structures, and work to enhance and facilitate the participation of a broad range of students in its doctoral programs.

APPENDIX

Interview outlines

I. Graduate students

The questions below are intended to give a direction to our discussion, but are not requirements for how it will develop. Feel free to delete anything from the list that you do not wish to discuss, and to add anything else that you feel might be relevant.

About your mathematical autobiography:

When did you first become interested in math? How did that interest develop?
What interested you about it? Have your interests changed?
Why do you like it?
What do you think mathematics is? Have these ideas changed?
Have there been any people who have been influential to you in mathematics?
What experiences have you had with mathematics, either in or out of school?
How did those experiences affect you?
Do you feel successful in mathematics?
Why did you decide to come to graduate school? Why at [the University]?

About your experiences in graduate school:

Which aspects of graduate school met your expectations? Which didn't? Why?

How did you like your classes?

How did they compare with your interests in math?

How do you learn math best?

What about other activities within the math department–how did they relate to your interests?

What have you enjoyed most about your experience here? What have you enjoyed least?

What have your relationships with professors been like? With other graduate students?

How would you describe your professors' beliefs about mathematics?

How do they compare with your own?

What does it take to succeed in graduate school?

If you are staying in the program:

Did you ever consider leaving? Why?

Which factors have been the most helpful in helping you to stay in the program, and to succeed to this point?

If you left the program, plan to leave, or are thinking of leaving without finishing:

At what point in your program were you when left or will you be when you leave?

When did you first start having doubts?

Why did you leave, will you leave, or are you thinking of leaving?

If you could change anything about the math department or program here to make it a better experience for you, what would you change?

Do you have any second thoughts?

What will you do/are you doing after graduate school?

Some other general questions:

Do you think gender, sexuality, race, ethnicity, nationality, age, handicap, or any other factors outside of your mathematical ability have had any impact on your experiences within mathematics? How?

Have these factors had any effect on the experiences of other graduate students?

What factors do you think influence the persistence or attrition of graduate students?

How would you design a math program if you really wanted to turn students on to math?

II. Faculty members

The questions below are intended to give a direction to our discussion, but are not requirements for how it will develop. Feel free to delete anything from the list that you do not wish to discuss, and to add anything else that you feel might be relevant.

About your mathematical autobiography:

When did you first become interested in math? How did those interests develop?

What interested you about it? Have your interests changed?

What is your mathematical autobiography? What was your relationship like with your advisor in graduate school? With other faculty? Who else was influential in your mathematical development?

How important was coursework in your professional development? What factors other than coursework were important to your professional development?

About your experiences as a mathematician:

What do you think mathematics is? Have these ideas changed over the course of your career?

What does it mean to be a mathematician?

How would you describe your graduate students' beliefs about mathematics? How do they compare with your own?

How would you describe your colleague's beliefs about mathematics? How do they compare with your own?

What is involved in doing mathematics? How did you learn to do these things?

About your role as a professor to graduate students:

What does it take for a graduate student to succeed?

What are the important aspects of the graduate student experience?

What is important for math graduate students to know, learn, and do? What is important in coursework?

When you teach graduate students, what are the things you emphasize?

How did you learn to teach? How important is teaching?

What do you do to prepare to teach a graduate course?

What is the purpose of qualifying exams, the specialty exam, courses, and other requirements?

About your role as an advisor:

What is your role as an advisor?

How does your view of your role as an advisor relate to the role of your own advisor?
How do you decide whether to work as an advisor to a particular graduate student? What do you look for?
How should a graduate student select an advisor?
What kinds of relationships should graduate students expect with advisors and with other faculty?
Is there anything the department could do to make a difference in your role as an advisor? In your relationships with graduate students in general?
Of your current graduate students, who is most likely to become a successful mathematical researcher (not necessarily by name)? Why?

Some other general questions:

What factors do you think are important in the persistence or attrition of graduate students from mathematics?
Do you think gender, sexuality, race, ethnicity, nationality, handicap, or any other factors outside of your mathematical ability have had any impact on your experiences within mathematics? In what ways?
Do you think any of these factors influence the experiences of graduate students within mathematics? How?
If you could change anything at all about the graduate program in mathematics to make a more successful or productive experience for graduate students, what would you change?

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NOTES

1. Two international students and two students enrolled in an interdisciplinary program were excluded. In addition, one U.S.-educated student was excluded because the issues he chose to explore in his interview were not relevant to the current study.
2. One of these continuing students who had doubts about completing his degree left the program some time after being interviewed. However, since this participant's intention was to continue at the time of the interview, he is included in the number of continuing students.

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Department of Learning and Teaching,
Graduate School of Education,
10 Seminary Place,
New Brunswick, NJ 08901, U.S.A.
E-mail: abbe.herzig@aya.yale.edu