WORKING TITLE: STUDENT MATHEMATICAL PERCEPTIONS AND SENSE-MAKING IN LIBERAL ARTS MATHEMATICS

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Abstract: A survey of views about the usefulness, processes, and nature of mathematics was administered twice in *Liberal Arts Mathematics* classes at a Doctoral I university. This was accompanied by interviews with students and instructors. Analysis of the survey and interview data suggests that the greatest impacts on student views on the time scale of one semester came from instructor style and number of years teaching experience. Interviews and observations of instructors found the prevalent teaching practices to be either Lecture-based or Activity-based. Among students in Activity-based courses perceived usefulness of mathematics and willingness to persist in solving moderately non-routine problems increased. In addition, level of student agreement with survey statements about the usefulness of mathematics and importance of sense-making decreased significantly in the classes taught by novices but went up among students of experienced teachers. The results illustrate the necessity of more thorough preparation of graduate teaching assistants before they teach college mathematics service courses.

Prepared 21 April 2004.

Introduction

Nurturing critical thinking skills in students, including the ability to use and trust their knowledge to solve novel problems, is essential for cultivating productive citizens who participate fully in a democratic society (George Report: Shaping the Future, 1996; NRC: Transforming Undergraduate STEM Education, 1999). The work reported here addresses a major challenge facing undergraduate mathematics education: identifying and expanding on ways to foster student learning in mathematics service courses. Here a "service course" refers to those courses taught in the department of mathematics to students who are not mathematics majors. These courses are usually referred to by students as "the last math course I'll ever take" ([author]) and include prospective elementary school teacher content courses, college algebra, finite mathematics, liberal arts mathematics, elementary statistics, business calculus, and other courses that satisfy general education/breadth requirements.

The quality of instruction for basic undergraduate mathematics courses is of concern across the United States (GET REF). National surveys indicate that fewer than 50% of full-time tenured faculty at four year institutions report teaching these courses (NCES Profile of Teaching, 2002). Most mathematics service courses are taught by new faculty with the least teaching experience; moreover, approximately 86% of graduate teaching assistants and part-time instructors under 35 years of age teach only undergraduate courses (NCES Profile of Teaching, p.14).

The larger research project, of which this report is a component, has at its heart the following question: *How do we increase student performance and sense-making in college mathematics while fostering college mathematics teaching effectiveness?* Building on the work of Boaler (1999) on concept-based teaching and learning at the high school level and Pajares' (1996) investigations of self-efficacy, the larger project contributes to the development of a model for analyzing self-efficacy in the context of first-year college mathematics. Here, the focus is on learning and teaching in liberal arts mathematics.

Theoretical Framework

The theoretical underpinnings for the study lie in a constructivist-informed approach to the social cognitive theory of human behavior framed by Bandura (1986). Social cognitive theory asserts that human achievement is part of a complex self-system consisting of cognitive, social, and affective knowledge structures which we use to filter our experiences, to understand what we learn, to make decisions, and to act. The ways humans observe, explore, and construct knowledge about the world include various forms of agentic self-regulation. *Agency* refers to acts done intentionally, with forethought and reflection (Bandura, 2001).

Within this framework, we use the phrase “flexible thinking” to indicate agentic work with one’s knowledge base. For thinking to be flexible, students need a content knowledge base with rich connections between ideas so that when one idea is triggered others follow, allowing several choices for response. The term *sense-making* refers to one’s efforts to construct understanding for oneself by organizing and coordinating the myriad information involved in a problem situation. This may involve, but is not restricted to, making sense of information in a textbook, from a lecture, or by working with problem situations in an activity-based class. Two main facets involved in sense-making are the depth of content knowledge and the breadth of application of such knowledge. Facility with both of these is a hallmark of flexible thinking in mathematics. Moreover, those with robust self-efficacy and self-regulatory abilities are able to continue to act intentionally and reflectively when faced with stress or anxiety that might accompany engagement with mildly non-routine problem situations (i.e., with any task where the solver does not already know a solution and is required to combine mathematical ideas in ways that may be new or novel for that solver (Selden, Selden, Hauk, & Mason, 2000)).

Whether or not a problem is experienced as non-routine depends on the perceptions and conceptions of the solver. To mathematics graduate students, for example, proof writing and proof validation tend to be non-routine mathematical problem situations. On the other hand, for a first-year undergraduate the tasks used in this study are likely to be non-routine. The reasons for using mildly non-routine problems in the study are twofold. First, novel problems are "real world" problems in the sense that most human experience, especially for young people, involves things never before encountered. Secondly, working on a mildly non-routine problem holds the potential to activate a wide array of cognitive and affective response whereas a routine exercise might be worked rather automatically, without bringing much to (or near to) the surface of conscious, expressible, thought. Key to understanding the interaction of cognition and affect is the triggering of such interaction, hence the non-routine problem-based interview protocol used in this study.

Acculturative stress

The discomfort, anxiety, numbness, even anger that many undergraduates report in connection with mathematics (Hall, 2002; Hauk, 2003; Tobias, 1993) may be related to the sociological construct of acculturative stress (Oyserman, Terry, & Bybee, 2002; Thompson, Anderson, & Bakeman, 2000). Acculturation is the process of revising one's self-view to include behaviors and personal views present in an environment where the dominant culture is different from one's own. The need to acculturate in relation to mathematics, in what may be perceived as a hostile mathematical environment, can cause physical, mental, and physiological tension and stress.

One way of resolving acculturative stress involves conceiving of a possible future self, or selves (Markus & Nurius, 1986), who can operate fluidly in both cultures. Work with adolescents and young adults indicates that imagining the self one could become in a new learning situation plays a key role in developing self-regulatory awareness for motivation and academic achievement (Leondari, Syngollitou, & Kiosseoglou, 1998; Oyserman et al., 2002). That is, resolving acculturative stress involves both affective and cognitive processes. Consequently, if the culture of mathematics is seen as alien (as it is by many undergraduates), achievement may be fostered by the "non-mathematical" articulation by students of their current self view and their envisioning of possible future selves persistent in mathematical problem solving. This is not to say that a mathematics course becomes a group therapy session. Rather, acknowledging, validating, and supporting student's acculturative efforts in mathematics may facilitate development of their mathematical sense-making abilities.

Acculturation to college teaching.

New college faculty, particularly graduate teaching assistants (GTAs), also wrestle with acculturation on many fronts. Among these are struggles to acculturate with their undergraduate students, with their new teaching peers, and with their department. Part of the acculturative process for new faculty is developing a flexible and adaptive form of cognitive structure that Ball and Bass (2000) have called *pedagogical content knowledge (PCK).* PCK is made up of *content knowledge* (e.g., knowledge of mathematics); *syntactic knowledge* about the nature and forms of communication and representation in mathematics; and *anticipatory knowledge* (or "knowledge for action") about the kinds of knowledge, views, and possible misconceptions that might arise as students engage with particular content. Instructors with rich PCK can teach in such a way that effective communication is maximized and misunderstanding and miscomprehension are minimized among their students. Environmental, behavioral, and personal factors all impact the development of effective teaching strategies and adaptive PCK. Novice college mathematics faculty typically have weakly structured PCK largely because of the lack of common experiences and views with their service-course students (McGivney-Burelle, DeFranco, Vinsonhaler, & Santucci, 2001).

Research Methods

The present study included an applied quantitative survey of the views held by liberal arts mathematics students about mathematics as well as analytic qualitative research on how undergraduates developed solutions in a non-routine problem situation during a task-based interview. The surveys aimed at large scale glimpses of student views of mathematics and their perceptions of themselves in doing and learning mathematics. A finer-detailed look at student views and at the interaction between their perceptions and their cognitions was the goal for the design and constant-comparative analysis of the task-based interviews.

Students

[Give demographics for university, for course, for classes.]

Primary data from students were the surveys and eight task-based interviews. Additional information was gathered from a review of student comments on the free-response prompt: "Comments (e.g...." [get exact wording) from the end-of-term course evaluations.]

Survey

Based on Grouws, Howald, and Coangelo's (1996) *Conceptions of Mathematics* inventory, an instrument was developed and piloted in one section of liberal arts mathematics in Summer 2002. Analysis of the pilot data indicated the instrument was both valid and reliable. In Fall 2002, all of the students in liberal arts mathematics (LAM) at the institution were surveyed.

The survey consisted of 35 items measuring five constructs: views about the purpose and usefulness or *Felt Needs* for algebra, geometry, and LAM; problem solving and verifying views about *Doing Mathematics*; and problem solving conceptual views about *Learning Mathematics*. Each item was a statement. Responses (circled on the survey by students) were on a six point scale: (1) strongly disagree, (2) disagree, (3) slightly disagree, (4) slightly agree, (5) agree, (6) strongly agree.

The survey was given in the first week of the course in nine liberal arts mathematics (LAM) classes to a total of 384 students. Completed surveys were returned by 375 students (a 97% completion rate). Survey administration was repeated in the last week of the course in seven sections in which 164 of 169 (97%) surveys distributed were completed (due to a scheduling mix-up, only one of Ms. Niles' classes did the post-course survey). The courses of one novice GTA and one lecturer were omitted from the analysis of survey data because post-course surveys were not collected in either of their classes.

Undergraduate Interviews

The survey instrument included an option for students to provide contact information if they were willing to be interviewed. In Spring 2003, eight students from among the 46 who had provided contact information on their surveys were interviewed by the first author. Interviewees were chosen to represent the broadest range of course grades and school years. The interview protocol asked students about their experiences in the LAM course, about other school and mathematical experiences, and included two tasks: one involving mildly non-routine problem solving and one involving problem posing (see Table 3). At least one student was interviewed from the classes of each of the five teachers for whom post-course data was collected. Interviews were conducted in accordance with institution-approved guidelines and protocols.

Instructors

Only the classes taught by Mr. Marsh, Mr. George, Ms. Niles, Ms. Dart, and Prof. Lee were involved in the both pre- and post-course survey. The sections taught by two other instructors, one novice GTA and one lecturer, were omitted from the analysis of survey data because they declined the administration of post-course surveys in their classes.

Instructors were asked about their educational philosophies and teaching practices. To lend perspective to student reports about their classes, the first author also observed at least one class for each instructor. Three instructors espoused and demonstrated mostly traditional views about college teaching practice and relied heavily on lecture and in-class quizzes for student assessment. These three Lecture-based instructors were Mr. George (1.25 years Full Time Equivalent (FTE) experience), Mr. Marsh (a novice college instructor), and Ms. Niles (more than 8 years FTE instructional experience). Two instructors asserted a constructivist or "reform" approach to teaching college mathematics that involved group-work during and outside of class meetings, projects, in-class quizzes that were sometimes cooperative or collaborative, and activities which might include posing and solving problems. These Activity-based instructors were Prof. Lee (more than 10 years FTE experience) and Ms. Dart (two years FTE experience). Table 1 summarizes the education, teaching style, and experience of the five instructors for whose classes full data were available.

Activity-based instruction uses activities as a primary method of teaching to create an active, engaging learning environment. This is not to say that Lecture-based instruction is not engaging. The primary difference between instruction type is the role of the instructor. Students do most of the talking and demonstrating about ideas in an Activity-based class, not the instructor. In a Lecture-based class the instructor's voice is most often heard and most physical or verbal demonstration is by the instructor. Though in common use, the phrases "learner-centered" and "teacher-centered" have been interpreted by some to mean, implicitly, that somehow one who lectures has no care, interest, or focus on learners. That is not our view. We have purposely chosen to describe pedagogical styles based on observable classtime activities in the hopes of avoiding such misinterpretation.

Pseudonym	Highest Degree in Mathematics	Teaching Style	Teaching Experience	# LAM sections Fall 2002
Mr. Marsh	M.S.	Lecture	none	1
Mr. George	M.S.	Lecture	1.25 years	1
Ms. Niles	M.A.	Lecture	8 years	2
Ms. Dart	M.S.	Activity	2.5 years	2
Prof. Lee	Ph.D.	Activity	10 years	2

Table 1. Information on instructors of the LAM classes.

Results

The survey

It is worth noting that attendance was low (compared to official enrollments) during the last week of the term, averaging 52% (23/44) for the Lecture-based sections and 73% (24/33) for the Activity-based sections. Consequently, of the 266 students officially enrolled in the sections of the course surveyed at the end of term, the 164 who did the survey represent an actual response rate of 62%. This difference in end-of-term class sizes and attendance levels is interesting. No notable differences on enrollment caps or times-offered existed for the two types of courses. However, the Lecture-based course instructors tended not to do informal advising of students whereas the Activity-based course instructors did – more on this below.

As mentioned, the survey had questions about *Felt Needs* views of algebra, geometry, and liberal arts mathematics (LAM) content, about *Learning Mathematics*, and about *Doing Mathematics*. In general, student reports did not change statistically significantly from start to end of course with two notable exceptions. Some reported *Felt Needs* views changed depending on the number of years of instructor experience and one reported *Doing Mathematics* view changed significantly among Activity-based students.

When Cronbach α reliability tests were run on the data we found $\alpha \geq .9$ for the Felt Needs about algebra, geometry and LAM constructs, $\alpha = .8$ for the views about Learning Mathematics construct, but $\alpha = .5$ for the views about Doing Mathematics construct. However, when reliability tests were computed *within instructor groups* – Lecture-based or Activity-based – we found $\alpha \geq .8$ for a subset of the construct within instructional approach. This calculation held true on post- *and pre-* course surveys even though the pre-course surveys were administered at the second class meeting. We conjecture that this was due to acculturation.

Acculturation of students to instructional style

The connection between instructional style (as suggested by the Cronbach α tests) and student views about the nature of doing and verifying mathematics seems to indicate a kind of sensitive dependence on initial conditions for student views. Dozens of students would have had to switch courses for an alignment of firmly held student views and instructional style to have occurred by the second day of the term. None of the instructors' or registrar's records indicate such large scale course-switching. We posit, therefore, that students were already in the process of attempting to take on the view of "doing mathematics" communicated by their instructor when they filled out the survey on the second day of class. That is, we may be seeing a kind of acculturative artifact: students may have learned to regulate their views and expectations about the nature of doing and verifying mathematical work, tailoring them to the espoused instructional and grading styles of their teachers. The fact that this self-regulation occurred by the second day of class is valuable information. More on this idea below.

Felt Needs. Student responses about the felt needs and purposes of algebra and geometry did not change significantly over the semester nor were there significant differences across instructional experience or style. The mean of responses about the usefulness of liberal arts mathematics in students' future work and citizenship maintained their averages from pre- to post-course but the variance among responses narrowed greatly. It could be argued that this is further support for the assertion made earlier that students tend to, at least for the semester, align their views with the expectations communicated by instructors.

The one area of LAM *Felt Needs* where student views changed was finances. ANOVA and post-hoc Tukey B tests indicated there was a significant difference between student response patterns pre and post course on the LAM items about finances ($p < 0.05$). Interestingly, this significant difference was found when *length of teaching experience*, regardless of style, was the controlling variable.

Instructor Experience The more developed an instructor's PCK, the more positive impact there appeared to be on student perceptions about the usefulness of LAM finances-oriented content. For instance, the students of the two instructors with eight or more years of full time equivalent (FTE) teaching experience, Ms. Niles and Prof. Lee, showed an increase in their agreement with a statement asserting that the mathematics in the LAM course had a lot to do with students' financial decisions, from 3.9 to 4.4, S.D. 0.45 (from slightly disagree to slightly agree). However, the students of novice instructors with three or fewer years FTE experience were more likely to disagree with that statement at the end of the course, from a mean of 3.9 to 3.4, S.D. 0.81. The difference between student response given levels of instructor experience was statistically significant ($p < 0.05$).

Also, the more experienced the teacher, the more students seemed to agree with statements about the need for autonomy in coming to understand mathematics. The least experienced instructors had a negative impact on the perceptions of their students. Though not statistically significant, it is worthwhile noting that the students of experienced instructors Ms. Niles and Prof. Lee were more likely to agree with statements about the importance of persistence and self-reliance than were the students of less experienced instructors.

Learning and Doing Mathematics. Student perceptions about how to *learn* and *do* mathematics, though different at the end of the course from at the beginning, were not significantly different, across years of experience and instructional style, with one exception. Students from Activity-based courses were statistically significantly more likely to disagree with the statement: "If you cannot solve a mathematics problem quickly, then spending more time on it won't help." Perhaps the actual classroom experience of fruitfully spending more time solving problems than they had in the past had an impact on Activity-based course students. This conjecture was supported by student interview results.

Undergraduate Interviews

Eight students were interviewed six months after the course was over. At least one student for each teacher was interviewed. Students' LAM grades are reflected in the first letter of pseudonyms (e.g. Annie had an A grade in LAM), see Table 2 for a summary of interview participants' information. The qualitative coding of interviews centered on four constructs: perceptions of the

LAM grade	Pseudo-nym	Sex	Age	Major area	College Class	College GPA	HS GPA	Math ACT	LAM Instr.	Lecture[L] Activity[A]
A	Annie	Fem	19	Music	Fresh	3.8	3.95	25	Dart	A
B	Beth	Fem	20	PoliSci &German	Fresh	3.8	3.7	17	George	L
B	Bud	Mal	19	Journalism	Soph	3.3	3.8	19	Lee	A
C	Cathy	Fem	19	Psychology	Soph	2.3	2.9	18	Niles	L
C	Cora	Fem	46	English	Junior	2.7	3.0	n/a*	Dart	A
D	Diana	Fem	24	History Ed.	Soph	2.5	2.5	n/a*	Marsh	L
D	Diego	Mal	20	Kines.& P.E.	Soph	1.6	2.5	n/a*	Lee	A
F	Felicia	Fem	19	Sociology	Soph	2.6	2.8	15	George	L

* Cora, Diana, and Diego were all transfer students for whom the Registrar had no score record.

Table 2. Student performance and personal data for the eight LAM interviewees.

course textbook, of teaching style, of the nature of mathematics, and of mathematical problem solving self-efficacy. Instructional style appeared to have interactions with each of the other three constructs to some extent.

The textbook. Student interview responses indicated that none of them was fond of their textbook. The text used by all of the lecturers was Johnson & Mowbry (2001); the text used by the Activity-based course instructors was Stazkow & Bradshaw (1995). The books are quite similar. The primary difference lies in the structure of problem-sets at the end of sections. The Stazkow &

Bradshaw book provides three sets of problems: Explain, Apply, and Explore. The first set are reading comprehension questions about the content of the section. The second set are fairly routine problems, the kind found in Johnson & Mowbry. The final set are mildly non-routine problems. Similar problems can be found scattered in Johnson & Mowbry, but they are not set-off as a separate kind of thinking and problem-solving.

During interviews, everyone except Annie said they rarely read the book (regardless of whether reading assignments were made) and that they typically only used it as "a last resort" (Diana) when "trying to figure out how to do the homework" (Bud). Students of Activity-based instructors reported reading and/or skimming the book more than the Lecture-based course students. It should be noted that both Activity-based course instructors included activities on how to read a mathematics book.

The students reported that they had gotten into the habit of looking at the homework and *then* referring to the text "only if you get stuck" (Felicia) when in middle school and early in high school mathematics. Each of them also said that it was hard to read a mathematics book. In fact, they all reported attempting to read the book at some point but "got hung up on the math parts, so I just skipped over them" (Bud). Reading a technical text, like a mathematics book, is not like reading a novel or a newspaper or an instruction manual. None of the eight students interviewed appeared to have acculturated much to mathematical reading.

Nature of mathematical learning and activity. On survey items that addressed facets of mathematical autonomy and self-efficacy there was a difference on post-course student response depending on instructional style that was not statistically significant. Six months after the course, mathematical self-efficacy perceptions reported during interviews varied and appeared not to be consistently correlated to the teaching style of a student's LAM instructor. The question arises, then, about the nature and persistence of the influence of LAM teachers' styles.

Students were asked about their LAM course experience. Bud commented on becoming more persistent, spending more time and effort on mathematical problems:

> Before I took that class [Activity-based LAM], considering the way I view organized math, I would never have said that spending more time on math was even something I would consider! But now, yeah.

Another Activity-based student, Cora, sighed and said “I’m just glad it’s over. I leave the math to my sisters and brother.” Diana noted that Mr. Marsh, “was one of those really super smart math guys and doesn’t really, like, explain down at other people’s level” while Beth commented on Mr. George’s apparent indifference to students’ views:

> Now, I don’t think it’s the teacher’s responsibility for me to learn anything, but I do think it’s a teacher’s responsibility to be sensitive to, ya know, that certain people don’t necessarily share the same [mathematical] lifestyle.

The students of instructors with several years experience said their instructors had communicated clearly to them some expectation of autonomy. Students felt responsible for their “performance in the class” (Beth, a student in a Lecture-based section) or for “making sense of it all, actually understanding” (Diego, in an Activity-based section). Students of less experienced instructors reported relying on instructor assertions of expertise to determine things like the relative need for autonomy and the usefulness of mathematics. One student, Diana, noted that she “didn’t really think it’s useful” but that in her LAM class she had been persuaded by the examples in her instructor’s lectures that he, Mr. Marsh, saw mathematics as useful. She said her survey response had remained “pretty positive” [in agreement with statements about the usefulness of mathematics] because she was “no expert. He showed us ways the stuff was useful. It makes sense to other people, I mean, ha, like I would ever be that way. But well, he’s the expert, so I went with what he said.” In other words, this student’s novice GTA instructor, in his efforts to negotiate the shoals of acculturation, may have relied heavily on asserting his own authority as a primary form of evidence for students to use in making decisions about the nature of mathematics. Mr. George’s students commented on his “I’m the authority” approach and both noted that he regularly remarked on how much "beneath" him teaching liberal arts mathematics was. Mr. Marsh and Mr. George were the two least experienced GTAs, both male and both Lecture-based instructors. Students' interview observations offer some support for the conjecture that the acculturative process for GTAs in their first year or so of learning to teach mathematics service courses can have a negative effect on their undergraduate students' building of autonomy in mathematics. Ms. Dart, the Activity-based GTA in her third FTE year of teaching, was also seen by her students as an authority, but not as authoritarian.

Problem situations. All eight students interviewed attempted the problem solving and problem posing tasks shown in Table 3.

Problem solving task: City Car Rental offers customers two methods of car rental. There is the Driver's Deal of \$42 per day with unlimited mileage at no extra charge. A second form of rental is the Run-about Rate of \$9.95 per day plus 10 cents per mile. If you wanted to rent a car from City Car Rental to go skiing (a roundtrip of three days with 400 miles of driving), which would be the least expensive for the trip: the Driver's Deal or the Run-about Rate? Justify your answer.
Problem posing prompt: Write a problem in mathematics based on a recent experience.

TABLE 3. LAM interview tasks.

Common to four of the eight students was their behavior upon being given the sheet containing the problem statements shown in Table 3: they physically distanced themselves from the sheet. Either the student pushed the paper away (Beth & Diana) or they pushed themselves away from the table upon which the sheet was lying (Diego & Felicia). Cora, a 46 year old woman returning to university after a messy divorce, turned the paper over a few times, both pulling it closer and pushing it away. The other three students, Annie, Bud, and Cathy, all came nearer the page, picking it up, pulling it to them, or hitching their chairs closer to the table. Students in the first group of four tended to report a view of mathematics (at least mathematics beyond simple whole number arithmetic) as something to be received from teachers. The last three students, Annie, Bud, and Cathy all tended to view mathematics as something over which they had some control and for the learning of which they assumed some responsibility for sense-making. Cora demonstrated traits from both groups, and several traits present in neither group (e.g. her actual arithmetic skills were at about the 5th grade level whereas all of the other students were functioning at 8th grade level or higher). Cora also appeared to be mathematically alexithymic (a reluctance or inability to articulate the results of conscious engagement in self-reflection and recall) where none of the seven others appeared so troubled. This is not to say she didn't *try* to express herself. She did. She was articulate about her children, her job, her English classes. However, whenever the conversation turned to her interactions with mathematics – from experiences in fourth grade to the previous semester – her sentences were suddenly incomplete and she became virtually incoherent.

Seven of the eight students' solutions to the problem-solving task were correct. The incorrect solution came from Felicia, the one student interviewed who had failed the course. For the problem-posing task, the three students able to clearly articulate a problem and provide a correct solution to it were from the Activity-based courses (only Cora was unable to clearly articulate a problem). The problems posed by these three, Annie, Bud, and Diego, all involved simple algebraic thinking and arithmetic. Each of these students was hesitant, at first, in creating a problem. However, each one became quite confident as they completed posing and moved on (without being prompted to do so by the interviewer) to making sense of and solving it. That is, the cognitions about what problem to pose and how to pose it and emotional response regarding their possible failure to do it correctly were well regulated enough to lead to a posed and solved problem. This is not really that surprising. Six months before the interview these students had been posing and solving problems regularly in their Activity-based LAM courses. The four students from the lecture-based courses evinced great trepidation when asked to pose a problem. Though each reported great relief when they were told the problem could be quite simple, thinking about what sort of problem to pose was "really hard." All four attempted to pose problems but either the problem itself was ill-posed (Felicia) or so poorly posed the student could not "figure it out. I know I wrote it, but I can't really make it make sense!" (Diana). None could correctly solve the problem she posed (Beth, Cathy, Diana), though Diana asserted quite forcefully that her answer was correct even though she could not articulate what the question was. The problems produced by these four students were complicated, involved lengthy formulas, and would have required multi-step methods to be solved. For these students from Lecture-based courses, affect and cognition and their interaction were not self-regulated effectively enough to complete the task. This suggests that these lecture-based course students had a different definition of what constituted a "problem" in mathematics and a different perception of their autonomy in mathematical situations from those students who had practice in problem-posing. Perhaps, then, practice in problem-posing in a Lecture-based course would be an effective means to increasing student ability to self-regulate the interaction between thoughts and feelings and would promote engagement in mathematical sense-making.

Influences on cognition and affect interaction dynamics

Survey results gave some indication of the interaction of student thoughts and feelings about learning, doing, and needing mathematics on the long-time scale of the semester while interview results provided some insight into the interaction of cognitive and affective components on the momentary scale of posing or solving a single problem. In between the two time scales lie the dynamics of class time interactions and their evolution over the weeks of a LAM course. Two of the influences on those dynamics may be, as was indicated in reporting the survey results, instructional style and length of experience. The presence of students in the classroom (e.g., attendance rates) was suggested by both Activity and Lecture-based instructors as an important factor. Students reported in their interviews that the classroom atmosphere promoted by the instructor "has a lot to do with whether you even want to show up to class" and that such ambiance was affected by "whether you think you are going to get anything out of it [being in class] that you couldn't get otherwise." Instructors and students also said (in various ways) that the preparedness of students for the course was a telling influence on how the class "feeling" and "work ethic" evolved over the course of the term.

Lecture-based style. Lecture-based course instructors reported one of two approaches to teaching the course. They asserted their goal was to "communicate the material" (George, Marsh, & Niles) and/or "to make it as painless as possible" (Marsh & Niles). These instructors agreed that communicating the material meant they allowed themselves to expect students to know only what had come out of the instructor's mouth. This very common stance in lower-division college teaching practice coincided with statements made by the departmental coordinator for the course at the Fall Coordination meeting. The coordinator made it clear that he saw the job of LAM instructors as "survey so they [the undergraduate students] can 'absorb and go'." Coordinator instructions included the following:

> This is a terminal course...meaning that this is quite likely the last math course the students are likely to take... Keep in mind that since this is a terminal course, you are not under pressure to finish all of the sections listed [on the suggested syllabus]. It is better to cover a topic well and skip some sections that students are bored with than to touch everything in a hurried manner.

Though the Lecture-based instructors acknowledged that mathematics may be an emotionally charged subject for their students, none of them seemed to fold this into PCK. Mr. George said he knew students got upset, but that they just had to "learn to get over it" and felt it was not his job, "not at all" to address their upset in any way. Mr. Marsh expressed deeply felt compassion for his students but used the descriptors "anxious," "lazy," and "sloppy" almost interchangeably in discussing student discomfort with mathematical situations. The highly experienced Lecture-based instructor Ms. Niles said she knew better “than to expect a lot, they just want to get through it and I help them do that.” Her goal was to set performance expectations and then, “Help them do well on quizzes and tests.”

The actual instructional techniques seen during observations of the Lecture-based courses coincided with the “absorb and go” goal of not taxing the students in LAM with much responsibility for autonomy in sense-making during class meetings. Common to all three instructors were regular appeals to an upcoming quiz or test as a motivation for students to attempt their homework. The more experienced instructors, Mr. George and Ms. Niles, also made it clear that they felt a student’s performance was a reflection of the student’s effort. Mr. Marsh, on the other hand, told his students that their recent poor performance on a quiz must be because he hadn’t communicated clearly enough and the class spent time going over the previous week’s material again. None of the Lecture-based instructors reported assessing student learning on all of the material suggested by the department’s course syllabus.

Activity-based style. Instructors of the Activity-based sections, Prof. Lee and Ms. Dart, had teaching approaches explicitly aimed at fostering both “critical thinking” and “mathematical calculation skills.” Both reported making overt and regular efforts to fold into classroom interactions the expectation that students were responsible for sense-making in coming to understand the mathematics being studied, whether or not the material had passed through the instructor’s mouth. The two Activity-based instructors also encouraged students to imagine themselves being persistent in the face of frustration, to allow the anxiety they felt to exist but to see themselves as continuing to work even when anxious. Though in conflict with departmental coordination expectations, the Activity-based instructional stance did coincide with research-mathematics’ cultural expectations about autonomy in learning and doing mathematics. The students in Activity-based courses might have benefited from these instructional choices in two ways: student perceptions about the usefulness of persistence in mathematics increased (statistically significantly in Prof. Lee's classes according to survey data) and their ability to engage in problem-posing and in solving moderately non-routine problems strengthened (as evidenced during task-based interviews).

Prof. Lee's classes "covered" and were assessed on the suggested material along with an additional chapter from the text (each of Lee's classes made their own choice about which chapter). Ms. Dart's classes did not complete all of the suggested material (by one chapter). Observations of Ms. Dart's classes indicated that though she was making her course Activity-based, when problems arose she would, as she put it, "fall back to lecture" as a way of "catching up." She reported doing this "a few times" in each of her classes.

Personality and instructional style. Defined as a characteristic way of thinking, feeling, and behaving, personality "embraces moods, attitudes, and opinions and is most clearly expressed in interactions with other people." It also "includes behavioral characteristics, both inherent and acquired, that distinguish one person from another and that can be observed in people's relations to the environment and to the social group." (E. Britannica, 2004). Personality certainly influences the way one teaches. Each of the instructional styles posited here was based on an instructor's choice about the form of her or his "interactions with other people." Administering personality tests to all of the instructors could have enriched our understanding of the facets of personality at work in the choice of teaching style. However, the focus of this study was the received curriculum: the experiences and perceptions of the students. The Lecture-based and Activity-based "instructional styles" used here arose from triangulation of the reports of students during interviews and on end-of-term evaluation forms, from instructors' self-descriptions, and from observations of classes. These reports all focused on the observable differences, particularly the locus of control for discussion, between the student-voice dominated Activity format and the instructor-voice dominated Lecture format.

Undergraduate Preparedness for LAM. Both Prof. Lee and Ms. Dart struggled with the diversity of ability levels present among students. Some were prepared for the course, Ms. Dart thought "about half of them." The rest were slightly to woefully under-prepared. This assessment of students was made possible through the activities done in class. Activities gave Ms. Dart and Prof. Lee the opportunity to move around the room to work with individuals and groups. Ms. Dart's comment prompted an examination of the college entrance mathematics scores (ACT or SAT) of LAM students.

Investigation through the Registrar's office of the academic background of a random sample of 36 of the 358 students who completed LAM for a grade in Fall 2002 revealed that at least half of them did not meet the minimum requirements set by the department for enrollment in the course. The department recommendation was that a student be enrolled in a 100-level mathematics course (rather than a remediation course) only if he or she had at least a 20/36 on the ACT-Math test, at least 500/800 on the SAT-Math, or at least three years of high school mathematics completed with a C or better. A score of 20 on the ACT-Math test was at the 55th percentile (i.e. 55% of students graduating high school scored at or below 20, the other 45% scored above 20) while a score of 18 was at the 32nd percentile (American College Testing, 2003). Eighteen of the 36 students in the sample met the requirement of 20 or higher. The other 18 students did not. Sixteen of these had ACT-Math scores below 18. The standard error (SE) of measurement for the ACT subject scores is ± 2 so a score below 18 is more than the SE away from the departmental recommendation of a score of 20.

As Prof. Lee and Ms. Dart observed, many of their students weren't ready for the course. Both Activity-based instructors acted on their observations early on, by advising individual students to take a remedial course, or to drop LAM if they did not have the time to spend on meeting the challenges of the course and to re-enroll when they could dedicate the time necessary, given their level of preparedness. This constituted the largest part of the informal advising by these instructors and accounted for most of the withdrawals shown in Figure 1.

Informal advising. Each LAM section started with approximately 45 students. On average, five students dropped each Activity-based class in the first week while two, on average, dropped out of each Lecture-based class. In total, forty-one students withdrew from their classes (i.e., dropped the course after the second week of classes). Of the 41 who withdrew, 36 dropped LAM and five switched to a different section of the course, with a different instructor. Four of these five section-switchers passed their classes. The distribution of withdrawls across instructors varied and appeared to be related to instructor teaching style and experience through informal advising.

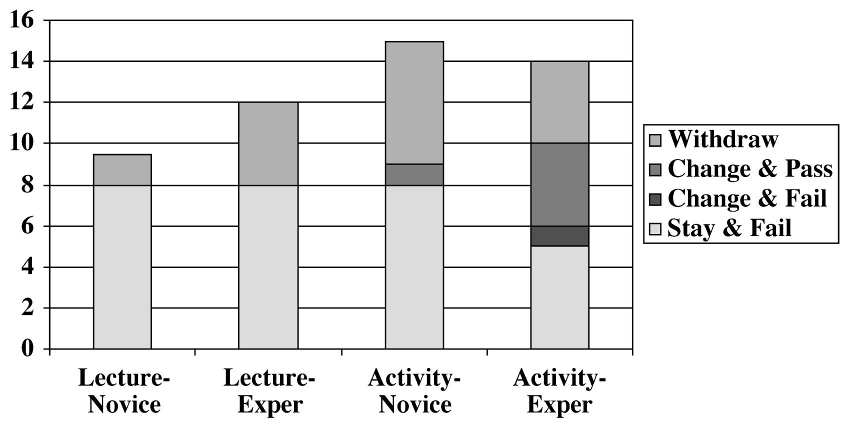

FIGURE 1. Averages of student course-leaving and failing, per class, by type of instructional style and level of experience.

The Activity-based instructors reported that the nature of interaction in their classrooms gave them the opportunity to consult individually with and informally advise students about their mathematics course-taking decisions during class time. Lecture-based instructors reported making general announcements in class such as “if you are struggling at this point, consider dropping the class” and reported few individual meetings with students that involved informal advising. As can be seen in Figure 1, the Activity-based instructors had a higher average number of withdrawals per class, and a higher number of students who switched sections (this was especially true for Prof. Lee, who was the experienced Activity-based instructor). Of interest in Figure 1 is the fact that the experienced Lecturer had an average of 8 students fail and 4 withdraw per class while the experienced Activity-based instructor had an average of 5 students fail, 4 withdraw, and 4 switch to another section and pass. Prof. Lee explicitly prompted students in the first week of the course to carefully consider course format and "comfort-level" in deciding whether or not to remain in the Activity-based section. Lee also remarked that most of the students who switched sections "made a good choice" and would likely have failed the Activity-based class because of their "reluctance to be active."

Grades in LAM. The average grades (across multiple sections if an instructor taught more than one) are illustrated in Figure 2. Again, we see the appearance of a pattern dependent upon instructional experience. The more experienced the instructor, the closer the average grade was to C. Neither Ms. Niles nor Prof. Lee (the two very experienced instructors) set up their classes in such a way that a normal distribution of grades was expected (i.e. there was no "curving" of scores). For Ms. Niles, the average grade of C arose in both of her sections from a large number of A and B grades being offset by a large number of F grades while for Prof. Lee, the distribution of grades was close to normal in both sections.

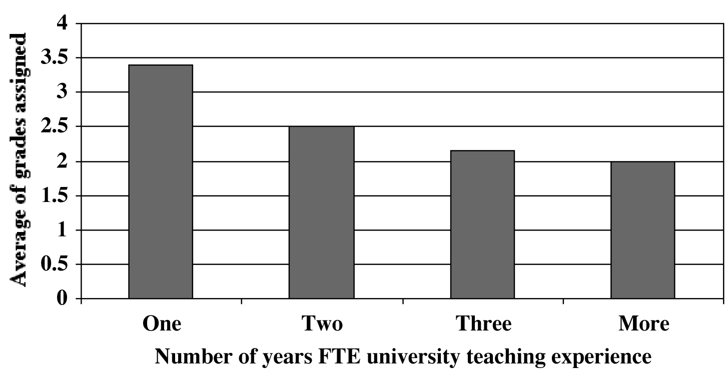

Figure 2. Average grade assigned, by instructor experience.

The less experienced instructors reported several grading challenges. Ms. Dart reported a few students who were loudly and repeatedly unhappy with the time and effort required by the course-assessment method of projects and the dearth of in-class exams. Mr. George and Mr.

Marsh reported negotiating “deals” for minimum performance on the final exam with students wherein a final exam score could be a student’s course grade if the score were higher than the student’s performance to date. In effect, the more junior Activity and Lecture-based instructors perceived themselves to be under pressure from their students to change their preferred teaching policies. Ms. Niles and Prof. Lee both perceived occasional student discontent. However, each had taught mathematics service courses for long enough to be able to place student comments in a larger experiential context than was possible for the less experienced teachers. That is, in the context of having interacted with over 1000 undergraduates, five complaints in a semester might be a relatively small thing. For a novice GTA, five complaints in a term from the first 40 students ever taught might loom quite large.

Conclusion

The original question motivating the study asked about the contents, strength, and persistence of student cognition and affect around non-routine mathematics. It would appear that for undergraduates in liberal arts mathematics the content of their conceptions about non-routine mathematics is susceptible to change, at least temporarily, depending on instructional style. In interviews, students in Activity-based courses asserted a conception of arithmetic and algebraic problems as mostly routine while Lecture-based course students tended to view ALL of mathematics as non-routine and often mystifyingly difficult. These conceptions were held strongly enough to persist and appear during interviews six months after their courses were over. Moreover, students of instructors with at least two years of experience perceived mathematics as more useful after their LAM course whereas students of inexperienced instructors viewed mathematics as less useful at the end of their courses. There was no strong evidence that student self-views had been significantly impacted by instructional style or years of experience on the broad scale of the survey. However, during interviews, comfort with mathematics – especially problem-posing – appeared to be much greater for the students from Activity-based courses.

What is the nature of the interaction of mathematical cognition and affect?

For undergraduates in liberal arts mathematics the answer appears to be: It depends. That is, the nature of the cognition-affect interaction depends on several factors including teaching style, teaching experience, and pedagogical content knowledge of the instructor. Students seem to align their expectations with those of their instructors for the cognitive and affective processes involved in doing and verifying mathematics.

When the LAM course was Lecture-based, affect and cognition interaction appeared to be counterproductive to sense-making. The Lecture-based instructors in this study reported that student affective response was something about which they were aware but with which they did not deal directly. Their students, in interviews, mirrored the separation of cognition and affect, reporting only that they "felt bad about feeling bad about math." This perceived separation may have contributed to students' inability to regulate the interaction of anxiety and "fear of failing" during attempts to activate effective cognitive effort during problem solving and posing.

Activity-based course instructors, on the other hand, directly and repeatedly addressed the anxiety and distaste students felt, validating student self-views and perceptions. They encouraged students to accept what they might feel, be it good, bad, or indifferent, and to concomitantly exert effort towards sense-making in their mathematical endeavors. Students in these courses exhibited the same align-with-the-instructor behavior as students in the Lecture-based courses. The question remains, for further research, about the longevity of such alignment of views.

Acculturative stress for students and teachers

Another facet of the study was the information garnered from and about GTAs as they gained experience as college instructors. It is important to keep in mind that as the undergraduate population grows more diverse, so do the graduate student and faculty populations. Though the dominant culture in academe is still white and middle-class, the fastest growing segment of the undergraduate population is neither (Delpit, 1996; Horn – NCES Profile of Undergraduates, 2002).

Already, rumblings of acculturative stress are being expressed by members of the Academy. These range from those in favor of new possible self views for the professoriate (Boyer, 1990; Boyer Commission, 1998; Fox, 2001; Tierney & Rhoads, 1994), to those in favor of maintaining the mid-20th century status quo (Gross, 1999; Hostetler, Sawyer, & Prichard, 2001). Key to all of these is the central role played by the graduate school experience in the development of a college professor (Boyer Commission, 1998). As diversity increases, the need for college instructors to monitor relationships among students in the classroom and intervene to address socio-cultural concerns (among other acculturative issues) also increases (Chism, 1988). That is, mathematics GTAs need practice and instruction in developing persistence and adaptivity in their efforts to communicate with non-mathematically enculturated undergraduates.

A shift towards autonomy in sense-making efforts is consonant with the expectations prevalent in constructivist-based reform movements in mathematics teaching and with academic mathematics culture, so it could be argued that students in Activity-based courses and in courses with experienced Lecturers become more mathematically bi-cultural. Also interesting was the finding that years of experience, moreso than teaching style, exerted influence on student perceptions of the usefulness of mathematics So, it may be that both a lecturer and a reform-oriented instructor can be "good college teachers," as long as they have had enough experience to build the acculturative adaptations in pedagogical content knowledge necessary for their teaching style to be effective with mathematics service course populations.

In this study, 45% of students in the LAM courses were prospective teachers, mostly from humanities, arts, and physical education areas. Pre-service K-12 teachers who come through mathematics service courses can be better served by more efficacious teachers, certainly. However, improving teaching is not enough. Improving the mathematical efficacy of students is also key. This study among liberal arts mathematics students suggests that perceptions of the usefulness of mathematics and conceptions about problem-solving and problem-posing can be positively influenced through college teaching aimed at both encouraging efficacious possible-future-self views and developing problem-posing ability in students.

Training new college faculty

Student views about *Doing Mathematics* contain both affective and cognitive components. The interaction between feelings and thoughts about the purpose and nature of doing mathematics appeared to be quite active in the first few days of the course. The fact that students aligned their reported views with those of their instructor by the second class meeting also points to the significance of messages communicated by the instructor on the first day of class. Research in behavioral science and memory indicates that first impressions are quite powerful (GET REF, (Kotre, 1995)). One path to improving teaching efficacy might be to focus new faculty attention on this fact. Given the *Doing Mathematics* survey results, preparation for new faculty aimed at unambiguous first-day-of-class communication certainly seems in order.

The study indicates that new faculty/GTA training should include preparing college mathematics teachers who are aware of and can act on the fact that though their students are adults, they may not share their instructor's mathematically enculturated views and may never have experienced mathematics as interesting or clear. Simultaneously, GTAs must come to grips with the environmental, behavioral, and personal factors at work within the departmental unit as they begin their content-area work. This double stress may be eased with training and support for GTAs that addresses the importance of structuring course coordination efforts to maximize pedagogical content knowledge development. Also, a future study might investigate the intentions of the instructors and the relationships between personality and instructional choice.

Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant Nos. DGE9906517 and DGE0203225. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation (NSF).

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